Optimal Steady-State Control for Frequency Regulation of Power Systems

John W. Simpson-Porco

UNIVERSITY OF WATERLOO

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This talk is based on these papers

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The Optimal Steady-State Control Problem

Liam S. P. Lawrence *Student Member, IEEE*, John W. Simpson-Porco, *Member, IEEE*, and Enrique Mallada *Member, IEEE*

Submitted to IEEE Transactions on Automatic Control

Optimal Steady-State Control for Linear Time-Invariant Systems

Liam S. P. Lawrence, Zachary E. Nelson, Enrique Mallada, and John W. Simpson-Porco

Paper & talk at IEEE Conf. Decision & Control (Miami, FL)

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Liam S. P. Lawrence
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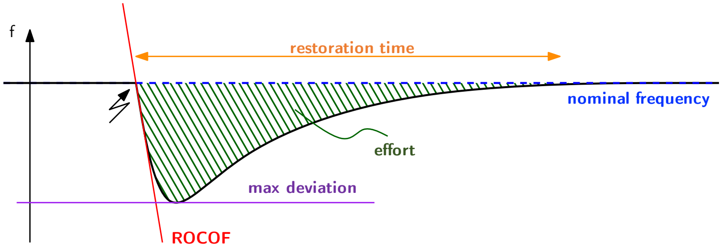
John's Hopkins Univ.

Zachary E. Nelson
Lockheed Martin

Frequency Control in Power Systems

Frequency changing $\longleftrightarrow$ power imbalance



1. **Small power system:** Integral control on frequency
2. **Big power system:** Automatic Generation Control (AGC)

$$\mathrm{ACE}_i = \beta_i(f_i - f_i^\star) + \sum_{(i,j)\in \text{Tie lines}} (p_{ij} - p_{ij}^\star)$$

Figure courtesy of F. Dörfler

Optimal Frequency Regulation Problem

1. Swing dynamics of **network of generators**

$$
\begin{aligned}
M_i\dot{\omega}_i &= P_i^* - D_i\omega_i - \sum\nolimits_{j\in\mathcal{E}} p_{ij} + u_i\,, && i \in \mathcal{N}\\
\dot{p}_{ij} &= b_{ij}(\omega_i - \omega_j)\,, && (i,j)\in\mathcal{E}
\end{aligned}
$$

2. Economically select equilibrium reserve powers u_i

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\begin{aligned}
\underset{u\in\mathbb{R}^n}{\text{minimize}} \quad & J(u) := \sum\nolimits_{i=1}^{n} J_i(u_i)\\
\text{subject to} \quad & \sum\nolimits_{i=1}^{n} (P_i^* + u_i) = 0
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- Constraint is **power balance** $\Longleftrightarrow$ **frequency regulation** $\omega = \mathbb{0}$
- Extensions: power limits, ramp rate limits, tie-line flow setpoints

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Optimal Distributed Frequency Regulation

A very incomplete literature review

Connecting Automatic Generation Control and Economic Dispatch from an Optimization View

Na Li, Lijun Chen,

Achieving Real-time Economic Dispatch in Power Networks via a Saddle Point Design Approach

Xuan Zhang, Na Li and Antonis Papachristodoulou

Distributed Frequency-Preserving Optimal Load Control

Enrique Mallada* Steven H. Low*

A unifying energy-based approach to optimal frequency and market regulation in power grids

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Reverse and Forward Engineering of Frequency Control in Power

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Underlying idea: Steady-state optimization with dynamics

What would a general problem formulation look like?

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General Control Problem Statement

Optimal Steady-State Control

Given data:

1. a dynamic system model w/ uncertainty specification
2. a vector of outputs $y \in \mathbb{R}^p$ of system to be optimized
3. a *class* of external disturbances $w(t)$
4. an optimization problem in y

Design, *if possible*, a controller such that

1. closed-loop is (robustly) well-posed
2. closed-loop is (robustly) stable
3. for all initial conditions, all disturbances within class, all possible uncertainties within specification

$$y(t) \longrightarrow \text{optimal value}$$

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Linear OSS Control: dynamics and achievable equilibria

1. **Uncertain LTI dynamics**

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\begin{aligned}
\dot{x} &= A(\delta)x + B(\delta)u + B_w(\delta)w \\
y &= C(\delta)x + D(\delta)u + Q(\delta)w \\
y_{\rm m} &= C_{\rm m}(\delta)x + D_{\rm m}(\delta) + Q_{\rm m}(\delta)w
\end{aligned}
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- δ = parametric **uncertainty**, w = const. **disturbances**
- $y_{\rm m}$ = system measurements available for **feedback**
- y = arbitrary system states/inputs to be **robustly optimized**

2. Affine set of **equilibrium values** for y

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\overline{Y}(w,\delta) = \underbrace{\mathrm{y}(w,\delta)}_{\text{offset vector}} + \underbrace{\mathrm{V}(\delta)}_{(\textit{unique})\text{ subspace}}
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Linear OSS Control: optimal steady-state

3. **Steady-state convex optimization problem**

$$\begin{aligned} y^\star(w,\delta) = \operatorname*{argmin}_{y \in \mathbb{R}^p} \quad & g(y,w) \\ \text{subject to} \quad & y \in \overline{Y}(w,\delta) = \mathrm{y}(w,\delta) + \mathrm{V}(\delta) \\ & Hy = Lw \\ & Jy \leq Mw \end{aligned}$$

- $y \mapsto g(y,w)$ is convex, **engineering** (in)equality constraints

4. gradient **KKT condition** for optimizer $y^\star$ is

$$\begin{aligned} & \nabla g(y^\star, w) + H^{\mathsf{T}}\mu^\star + J^{\mathsf{T}}\nu^\star \quad \perp \quad \boldsymbol{V(\delta)} \\ \iff \quad & \nabla g(y^\star, w) + J^{\mathsf{T}}\nu^\star \quad \perp \quad (\boldsymbol{V(\delta)} \cap \textbf{ker } \boldsymbol{H}) \end{aligned}$$

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When can we optimize robustly?

Problem: model uncertainty δ enters KKT conditions

$\implies$ we cannot design δ-independent controller!

When can KKT be **robustly** (i.e., $\forall \delta \in \boldsymbol{\delta}$) enforced?

1. **Robust Output Subspace** (ROS) property

$$\mathsf{V}(\delta) \text{ is independent of } \delta$$

2. **Robust Feasible Subspace** (RFS) property

$$\mathsf{V}(\delta) \cap \ker H \text{ is independent of } \delta$$

If **either** of these properties hold, can **robustly** enforce orthogonality in gradient condition via an **optimality model**

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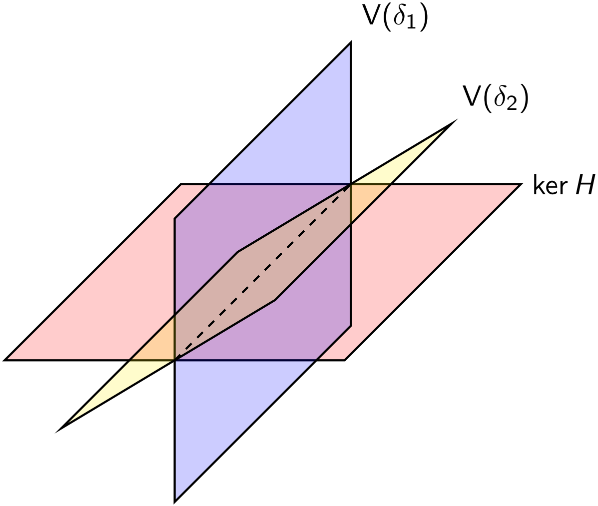
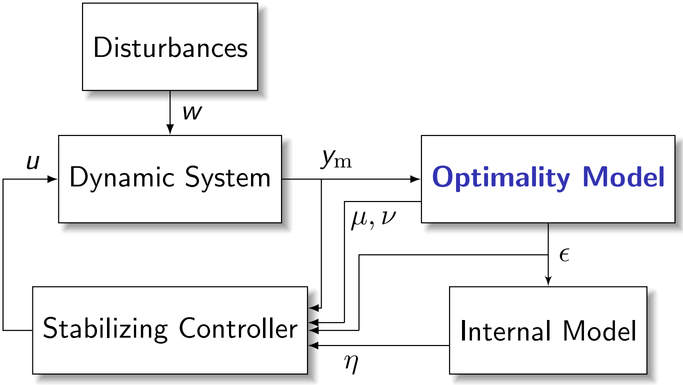
The Robust Feasible Subspace Property



General Design for Linear OSS Control



An **optimality model** is a dynamic system which **robustly** produces a **proxy** ϵ for optimality error

Optimality Models for Linear OSS Control

1. Robust Output Subspace (ROS) Optimality Model

$$\dot{\mu} = Hy - Lw$$
$$\dot{\nu} = \boldsymbol{max}(\nu + Jy - Mw, \mathbb{0}) - \nu$$
$$\epsilon = \boldsymbol{R_0}^\mathsf{T}(\nabla g(y, w) + H^\mathsf{T}\mu + J^\mathsf{T}\nu)$$

$$\text{range } R_0 = \mathsf{V}(\delta)$$

(Design freedom!)

2. Robust Feasible Subspace (RFS) Optimality Model

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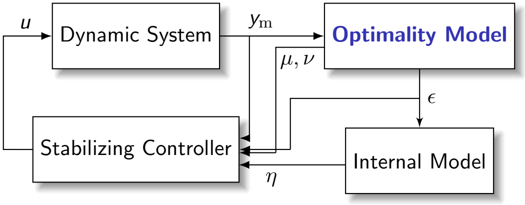
Internal Model and Stabilizer Design



For **constant** disturbances, internal model is **integral control**

$$\dot{\eta} = \epsilon$$

Stabilizer design options:

1. high-gain feedback of ϵ (minimum phase systems)
2. full-order dynamic robust controller synthesis
3. low-gain integral control $u = -k\eta$ (E. J. Davison 1976)
4. problem-specific judgement

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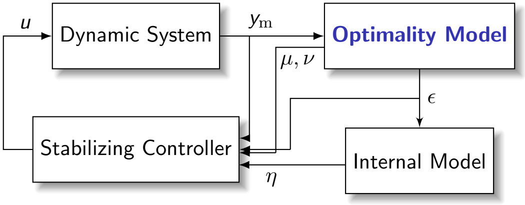

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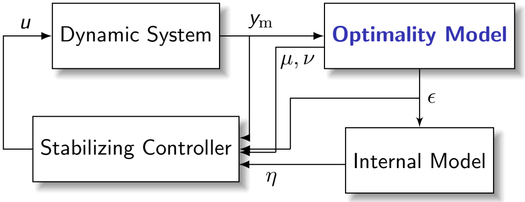

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Equivalent optimization problems will lead to **different** OSS controllers (more design flexibility)

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OSS Framework Recovers Standard Controllers

1. Distributed-Averaging Proportional-Integral Control [JWSP et. al. '12]

$$\dot{p}_i = \omega_i - \sum_{j \in \mathcal{N}} a_{ij}(\nabla J_i(p_i) - \nabla J_j(p_j)) , \qquad u_i = -p_i$$

2. Gather-and-Broadcast Control [Dörfler & Grammatico, '17]

$$\dot{\mu} = \text{average}(\omega_i) , \qquad u_i = (\nabla J_i)^{-1}(\mu)$$

3. Primal-dual algorithm [Li, Zhao, Mallada, Topcu, Low, ...]

$$\dot{\mu}_i = -\nabla J_i(\mu_i) - \nu , \qquad u_i = \mu_i$$
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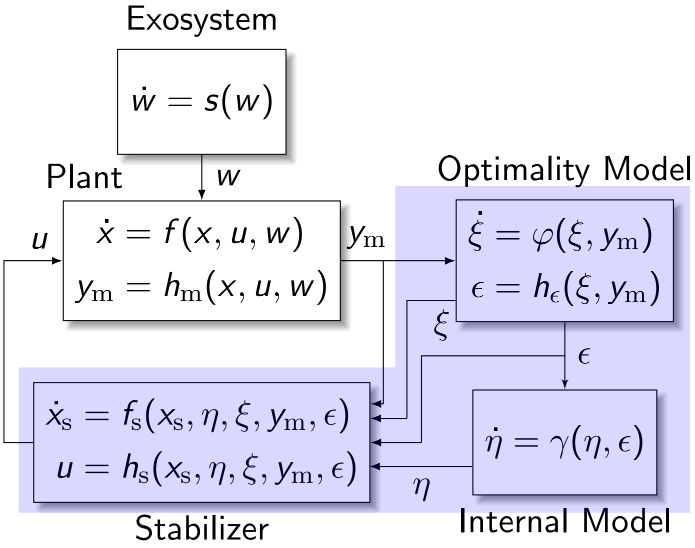
Nonlinear OSS Control Problem Architecture

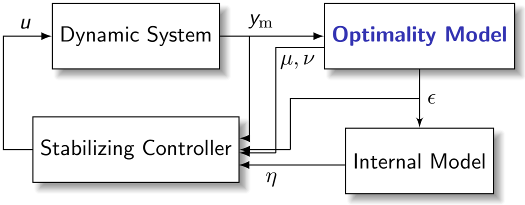
Nonlinear systems with **time-varying** disturbances



Conclusions

New control framework: Optimal Steady-State (OSS) Control

1. Optimize dynamic systems robustly w.r.t. uncertainty/disturbances
2. Ensure dynamic performance and robustness



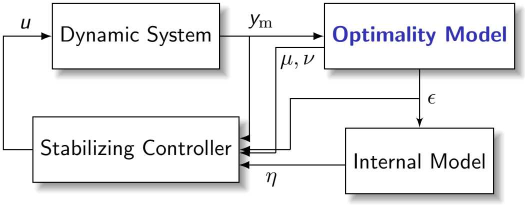
What's next?

1. Sampled-data, decentralized, hierarchical OSS control
2. Detailed application case studies

Conclusions

New control framework: Optimal Steady-State (OSS) Control

1. Optimize dynamic systems robustly w.r.t. uncertainty/disturbances
2. Ensure dynamic performance and robustness



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Details in our new papers (available on my website)

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The Optimal Steady-State Control Problem

Liam S. P. Lawrence *Student Member, IEEE*, John W. Simpson-Porco, *Member, IEEE*, and Enrique Mallada *Member, IEEE*

Submitted to IEEE Transactions on Automatic Control

Optimal Steady-State Control for Linear Time-Invariant Systems

Liam S. P. Lawrence, Zachary E. Nelson, Enrique Mallada, and John W. Simpson-Porco

Paper & talk at IEEE Conf. Decision & Control (Miami, FL)

Questions

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appendix