Delft Center for Systems and Control

Coordination of Energy Supply and Demand

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Tutorial: "Distributed Control and Optimization for Autonomous Power Grids"

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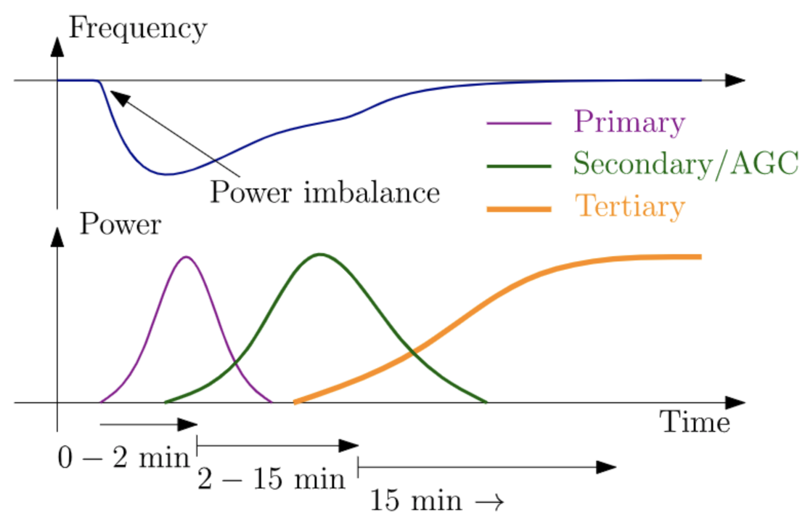
Outline

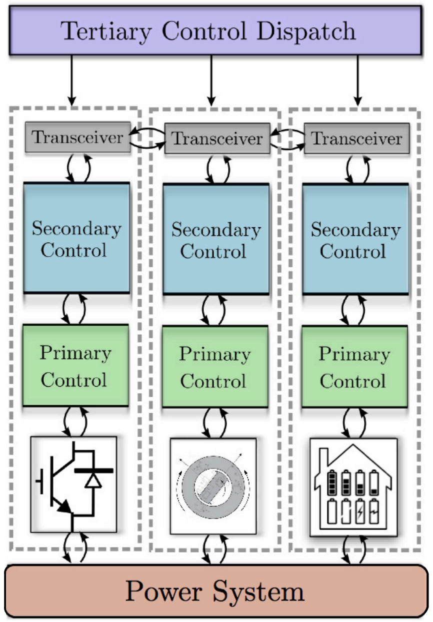
Outline

From Secondary to Tertiary Control in Power Systems



Ersdal et al., *Model predictive load-frequency control taking into account imbalance uncertainty* (Fig. 1), IEEE CEP, 2016

From Secondary to Tertiary Control in Power Systems (+)



3. **Tertiary** control (**offline**)
   - optimize operation
   - power scheduling/dispatch

2. Secondary control (slow real-time)
   - reference tracking
   - centralized/distributed
   - integral control (AGC)

1. Primary control (fast real-time)
   - local asymptotic stability
   - decentralized
   - proportional control (droop)

Courtesy: F. Dörfler

From Secondary to Tertiary Control in Power Systems (++)

(Simplified) **Power Systems dynamics**

$$\begin{aligned} \Delta\dot{\theta}_i &= \Delta\omega_i \\ M_i\,\Delta\dot{\omega}_i &= -D_i\Delta\omega_i + \Delta P_i + u_i - \textstyle\sum_j B_{i,j}\sin(\Delta\theta_i - \Delta\theta_j) \end{aligned}$$

- u_i = controlled power injection

Optimal Economic Dispatch

$$\begin{cases} \min\limits_{\{u_i^{\text{ref}}\}_i} & \sum_i J_i(u_i^{\text{ref}}) \\ \text{s.t.} & u_i^{\text{ref}} \in \{\text{limits}\},\ \forall i \\ & \sum_i \Delta P_i^{\text{ref}} + u_i^{\text{ref}} = 0 \quad \longleftarrow \textbf{power balance} \end{cases}$$

- (goal) $\forall i : \lim\limits_{t\to\infty} u_i(t) = u_i^{\text{ref}\star}$ = optimal solution

Some Literature on Secondary/Tertiary control

- Simpson-Porco, Dörfler, Bullo, *Synchronization and power sharing for droop-controlled inverters in islanded microgrids*, Automatica, 2013
- Li, Zhao, Chen, *Connecting automatic generation control and economic dispatch from an optimization view*, IEEE TCNS, 2015
- Dörfler, Simpson-Porco, Bullo, *Breaking the hierarchy: Distributed control and economic optimality in microgrids*, IEEE TCNS, 2016
- Mallada, Zhao, Low, *Optimal load-side control for frequency regulation in smart grids*, IEEE TAC, 2017
- Cai, Mallada, Wierman, *Distributed optimization decomposition for joint economic dispatch and frequency regulation*, IEEE TPS, 2017
- Dörfler, Grammatico, *Gather-and-broadcast frequency control in power systems*, Automatica, 2017
- Trip, Cucuzzella, De Persis, Van der Schaft, Ferrara, *Passivity-based design of sliding modes for optimal load frequency control*, IEEE CST, 2018
- Stegink, Cherukuri, De Persis, Van der Schaft, Cortés, *Hybrid interconnection of iterative bidding and power network dynamics for frequency regulation and optimal dispatch*, IEEE TCNS, 2018

Tertiary Control and Electricity Markets

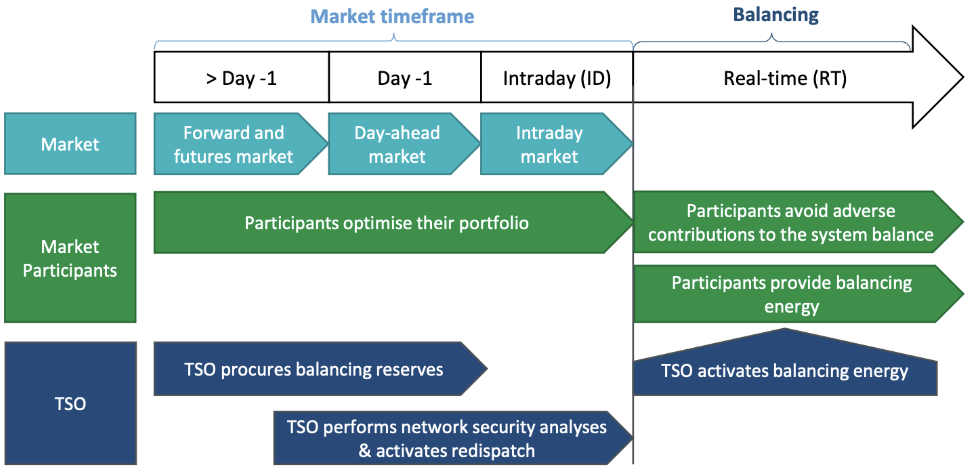
Optimal Economic Dispatch $\Longrightarrow$ **Cooperative** optimization

Nowadays:

- Distributed generation & flexible prosumption
- Retailers: Aggregation of small prosumers
- Deregulated electricity markets

$\Rightarrow$ Generators operated by **Competing**/**Non-Cooperative** firms

Wholesale Electricity Markets

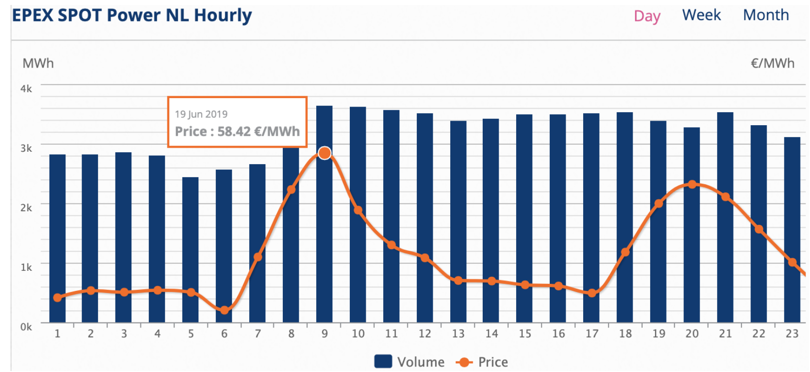


TenneT, Dutch Transmission System Operator (TSO), Annual market update 2018, `tennet.eu`

Day ahead and Intraday Electricity Markets

Examples of **Day-Ahead/Intraday Electricity Markets**:

- European Power Exchange (EPEX) [DE, FR, UK, ..., NL, CH]
- Amsterdam Power Exchange (APX) [NL, BE, UK]



- Gestore Mercati Energetici (GME) [IT]
- Nord Pool [UK, NO, SE, FI, ...]

Day ahead and Intraday Electricity Markets

Generators:

$$(\forall i) \quad \mathbb{P}_i : \begin{cases} \max\limits_{s_i, u_i} & \underbrace{p(\textstyle\sum_j s_j)}_{\text{nodal prices}}{}^\top s_i \; - \; \underbrace{c_i(u_i)}_{\text{generation cost}} \; - \; \underbrace{\operatorname{diag}(\lambda)(s_i - u_i)}_{\text{fees}} \\ \text{s.t.} & (s_i, u_i) \in \{\text{limits}\} \end{cases}$$

- s_i = sale, u_i = generation (at all nodes)
- λ = transmission fees

TSO/ISO:

$$\mathbb{P}_{\text{iso}} : \begin{cases} \max\limits_{\lambda} & \text{revenue}(\lambda, \boldsymbol{s}, \boldsymbol{g}) \\ \text{s.t.} & (\boldsymbol{s}, \boldsymbol{u}) \in \{\text{transmission capacities}\} \end{cases}$$

Some Literature on Electricity Markets

- Hobbs, Metzler, Pang, *Strategic gaming analysis for electric power systems: An MPEC approach*, IEEE TPS, 2000
- Day, Hobbs, Pang, *Oligopolistic competition in power networks: a conjectured supply function approach*, IEEE TPS, 2002
- Niu, Baldick, Zhu, *Supply function equilibrium bidding strategies with fixed forward contracts*, IEEE TPS, 2005
- Hobbs and Pang, *Nash–Cournot equilibria in electric power markets with piecewise linear demand functions and joint constraints*, Operations Research, 2007
- Conejo, Carrion, Morales, *Decision making under uncertainty in electricity markets*, Springer, 2010
- Gabriel, Conejo, Fuller, Hobbs, Ruiz, *Complementarity modeling in energy markets*, Springer, 2012
- Morales, Conejo, Madsen, Pinson, Zugno, *Integrating renewables in electricity markets: Operational problems*, Springer, 2014

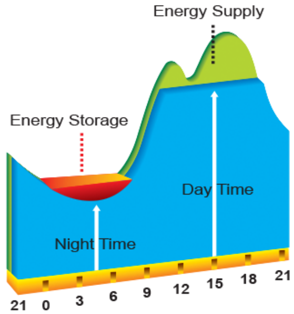
Local Day ahead Market: Demand Side Management

Demand Side Management:

de-synchronize / flatten net energy demand of prosumers

Prosumer i:

$$\begin{cases} \min_{u_i} \ \text{discomfort}_i(u_i) \\ \qquad + \text{congestion\_cost}(u_i, \underbrace{\textstyle\sum_j u_j}_{\text{aggregation}}) \\ \text{s.t.} \ \ u_i \in \{\text{limits}\} \end{cases}$$



Mohsenian-Rad et al., *Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid*, IEEE TSG, 2010

Saad, Han, Poor, Başar, *Game-theoretic methods for the smart grid*, IEEE MSP, 2012

Outline

Cooperative Balancing & Duality

Optimal Economic Dispatch:

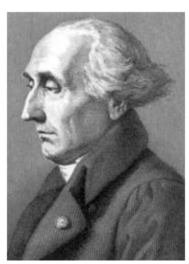
$$\mathbb{P}_0 : \begin{cases} \min_{\{u_i^{\text{ref}}\}_i} & \sum_i J_i(u_i^{\text{ref}}) \\ \text{s.t.} & u_i^{\text{ref}} \in \{\text{limits}\}, \forall i \\ & \sum_i \Delta P_i^{\text{ref}} + u_i^{\text{ref}} = 0 \quad \longleftarrow \text{power balance} \end{cases}$$

(ease notation: drop $^{\text{ref}}$)

Lagrangian function:

$$L(\boldsymbol{u}, \mu) := \textstyle\sum_i \{J_i(u_i) + \iota_i(u_i)\} + \mu^\top \sum_i \{\Delta P_i + u_i\}$$

- ι_i = *indicator function* for $u_i \in \{\text{limits}\}$



Cooperative Balancing & Duality (+)

KKT Theorem: $\boldsymbol{u}$ solves $\mathbb{P}_0$ iff (for some μ)

$$\text{KKT}_0 : \begin{cases} \mathbb{0} \ \in \ \partial_{\boldsymbol{u}} L(\boldsymbol{u}, \mu) & \longleftarrow \text{stationarity} \\ 0 \ = \ \sum_i \Delta P_i + u_i & \longleftarrow \text{feasibility} \end{cases}$$

Separability:

$$L(\boldsymbol{u}, \mu) = \textstyle\sum_i \underbrace{J_i(u_i) + \iota_i(u_i) + \mu^\top (\Delta P_i + u_i)}_{L_i(u_i, \mu)} = \sum_i L_i(u_i, \mu)$$

$$\implies 0 \in \partial_{u_i} L(\boldsymbol{u}, \mu) = J_i'(u_i) + \underbrace{\partial \iota_i(u_i)}_{\text{normal cone}} + \mu$$

$$\implies u_i = \text{sat}\left(-J_i'^{-1}(\mu)\right)$$

Cooperative Balancing & Duality (++)

- $u_i(\mu) = \text{sat}\left(-J_i'^{-1}(\mu)\right)$

From Convexity to Monotonicity:

- J_i strictly convex

$\Rightarrow$ J_i' strictly increasing $\Rightarrow$ $J_i'^{-1}$ strictly increasing

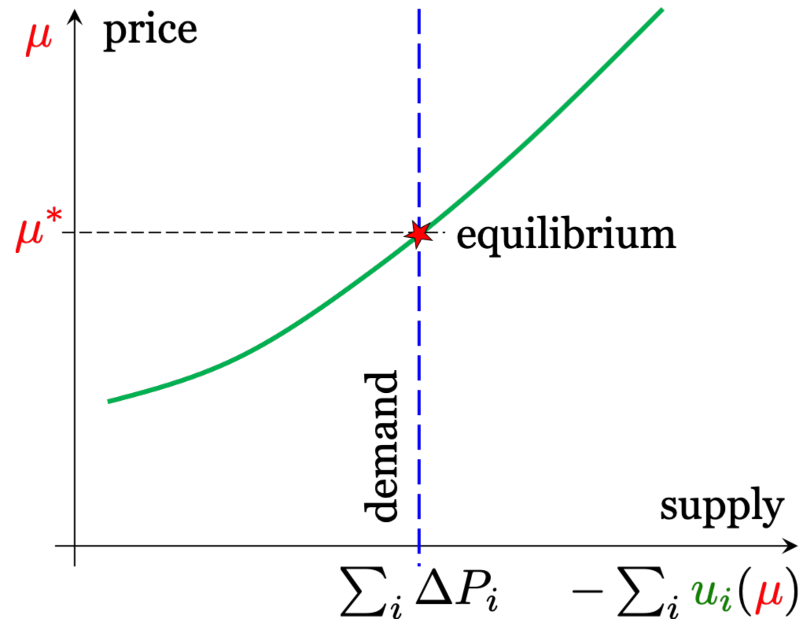
$\Rightarrow$ $-J_i'^{-1}$ strictly decreasing $\Rightarrow$ $\text{sat}(-J_i'^{-1}(\cdot))$ strictly decreasing

μ = **Imbalance price**:

$$\mu \nearrow \quad \Longrightarrow \quad u_i(\mu) \searrow \quad \Longrightarrow \quad \underbrace{\textstyle\sum_i \Delta P_i + u_i(\mu)}_{\text{imbalance}} \searrow$$

(Market) Clearing: $\sum_i \Delta P_i + u_i(\mu^*) = 0$

Cooperative Balancing & Duality (+++)



Cooperative Balancing & Duality (multi-stage)

$$\mathbb{P}_0 : \begin{cases} \min_{\{u_i\}_i} & \sum_i J_i(u_i) \\ \text{s.t.} & u_i \in \mathcal{U}_i, \ \forall i \\ & \sum_i u_i = \mathbb{0} \quad \longleftarrow \text{ multi-stage } \textbf{balance constraint} \end{cases}$$

- u_i $(\in \mathbb{R}^n)$ = *deviation* from nominal reference over time

Lagrangian function $L(\boldsymbol{u}, \boldsymbol{\mu})$ still **separable** ☺

- $\boldsymbol{\mu}$ $(\in \mathbb{R}^n)$ = *imbalance prices* over time

Non-Cooperative Balancing

Electricity market: each generator shall minimize its own cost

$$(\forall i) \ \mathbb{P}_i(\boldsymbol{u}_{-i}) : \begin{cases} \min_{u_i} & J_i(u_i, \boldsymbol{u}_{-i}) \quad \longleftarrow \textbf{coupled cost} \\ \text{s.t.} & u_i \in \mathcal{U}_i \\ & u_i + \sum_{j \neq i} u_j = \mathbb{0} \quad \longleftarrow \textbf{balance constraint} \end{cases}$$

- $\boldsymbol{u}_{-i} := (u_j)_{j \neq i}$

Generalized Nash Equilibrium (GNE):

$\boldsymbol{u}^* = (u_i^*)_i$ s.t.

$$\forall i : \ u_i^* \text{ solves } \mathbb{P}_i(\boldsymbol{u}_{-i}^*)$$

$\rightarrow$ each decision is optimal given the others

Non-Cooperative Balancing & Duality

Lagrangian functions:

$$L_i(u_i, \boldsymbol{u}_{-i}, \mu_i) := J_i(u_i, \boldsymbol{u}_{-i}) + \iota_i(u_i) + {\mu_i}^\top \left(u_i + \textstyle\sum_{j \neq i} u_j \right)$$

KKT Theorem: u_i solves $\mathbb{P}_i(\boldsymbol{u}_{-i})$ iff (for some $\mu \in \mathbb{R}^n$)

$$(\forall i) \quad \mathrm{KKT}_i : \begin{cases} \mathbb{0} \in \partial_{u_i} L_i(u_i, \boldsymbol{u}_{-i}, \mu) & \longleftarrow \text{stationarity} \\ \mathbb{0} = u_i + \sum_{j \neq i} u_j & \longleftarrow \text{feasibility} \end{cases}$$

- $\partial_{u_i} L_i = \nabla_{u_i} J_i(u_i, \boldsymbol{u}_{-i}) + \partial \iota_i(u_i) + \mu$

Main Problem: How to solve the interdependent KKT systems?

Coordinated Best Response algorithm

Iterative bidding (index $k \in \mathbb{N}$):

$$u_i(k+1) = \operatorname{argmin} L_i(\cdot, \boldsymbol{u}_{-i}(k), \mu(k)) \quad \longleftarrow \textbf{best response}$$

$$\mu(k+1) = \mu(k) - \epsilon \textstyle\sum_j u_j(k+1) \quad \longleftarrow \textbf{price adjustment}$$

Convergence (strong convexity/monotonicity, small step, ...):

$\lim_{k \to \infty} \boldsymbol{u}(k) = \boldsymbol{u}^*$ **GNE**

- Best response updates run in **parallel**
- Price adjustment requires **aggregate** information

Coordinated Projected Gradient algorithm

Iterative bidding (index $k \in \mathbb{N}$):

$$u_i(k+1) = \operatorname{proj}_{\mathcal{U}_i} \left[u_i(k) - \epsilon (\nabla_{u_i} J_i(u_i(k), \boldsymbol{u}_{-i}(k)) + \mu(k)) \right]$$

$\longleftarrow$ **projected gradient step**

$$\mu(k+1) = \mu(k) - \epsilon \textstyle\sum_j 2u_j(k+1) - u_j(k) \quad \longleftarrow \textbf{price adjustment}$$

Convergence (strong convexity/monotonicity, small step, ...):

$\lim_{k \to \infty} \boldsymbol{u}(k) = \boldsymbol{u}^*$ **GNE**

- Projected gradient step replaces local best response

Outline

Coordinated Bidding as Fixed-Point Iteration

Coordinated Best Response:

$$\begin{bmatrix} u_1(k+1)\\ \vdots \\ u_N(k+1)\\ \mu(k+1)\end{bmatrix} = \underbrace{\begin{bmatrix} & \mathrm{Id} & & \\ -\epsilon I & \cdots & -\epsilon I & \mathrm{Id}\end{bmatrix} \circ \begin{bmatrix} \ddots & & & \\ & \mathrm{argmin}\, L_i & & \\ & & \ddots & \\ & & & \mathrm{Id}\end{bmatrix}}_{=:T} \begin{bmatrix} u_1(k)\\ \vdots\\ u_N(k)\\ \mu(k)\end{bmatrix}$$

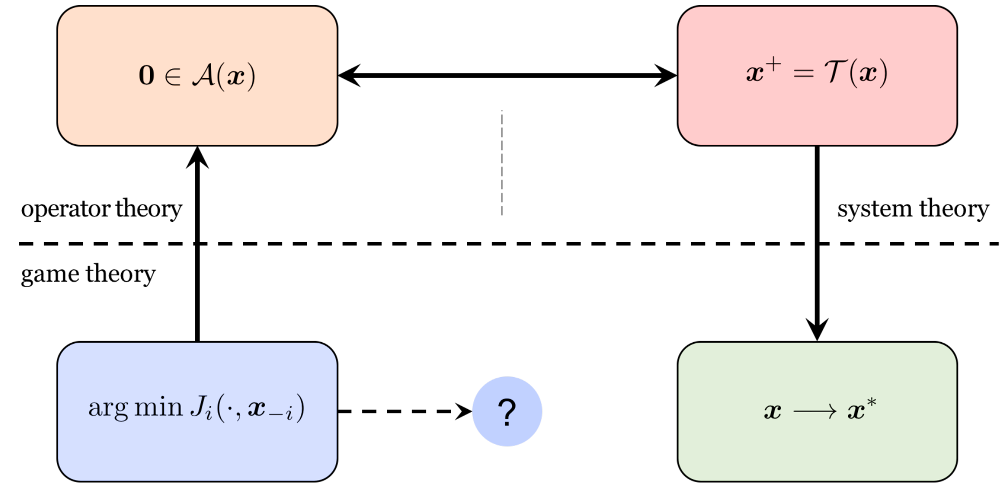
From Zeros to Fixed Points:

$(\boldsymbol{u}^*, \mu^*)$ solves $\{\mathrm{KKT}_i\}_i \iff (\boldsymbol{u}^*, \mu^*) \in T(\boldsymbol{u}^*, \mu^*) \longleftarrow$ **fixed point**

Theorem:

T **Averaged** (e.g. **Contraction**) $\Longrightarrow$ **Convergence** to **Fixed Point**

From Monotone Operator Theory to Fixed-Point Iterations



Bauschke, Combettes, *Convex analysis and monotone operator theory in Hilbert spaces*, 2nd ed., SPRINGER, 2016

Outline

Conclusion

- Tertiary Control in Power Systems $\longmapsto$ Equilibrium Problem
- (Non-Cooperative) Balancing of Energy Supply & Demand

  $\longmapsto$ Generalized Nash Equilibrium Problem
- Solution Architecture: Iterative Bidding & Price Adjustments

Outlook

- Interconnection of

  Power Network Dynamics & Equilibrium Seeking Algorithms

- **Incremental Passivity** is key

- Gadjov, Pavel, *A passivity-based approach to Nash equilibrium seeking over networks*, IEEE TAC, 2019
- De Persis, Monshizadeh, *A feedback control algorithm to steer networks to a Cournot–Nash equilibrium*, IEEE TCNS, 2019
- Pavel, *On incremental passivity in network games*, ISDG NetGCoOp, 2019

Thank you for your kind attention

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