Optimal and Distributed Frequency Control of Transmission Grids

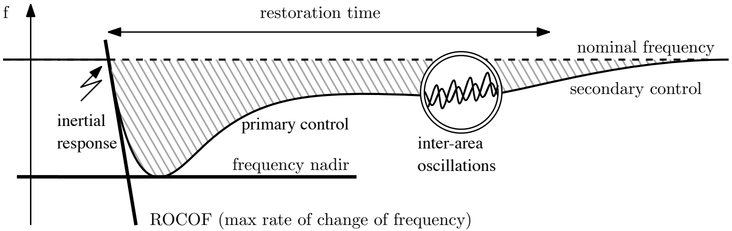
John W. Simpson-Porco

UNIVERSITY OF WATERLOO

ECC Tutorial Session, Naples, Italy

June 27, 2019

Response to disturbance of an **ideal** power system



1. Physical inertia instantly provides decentralized **derivative** control
2. Decentralized *primary loops* at devices provide **proportional** control

Disturbance *attenuation*

3. System-wide *secondary loop* provides centralized **integral** control

Disturbance *rejection*

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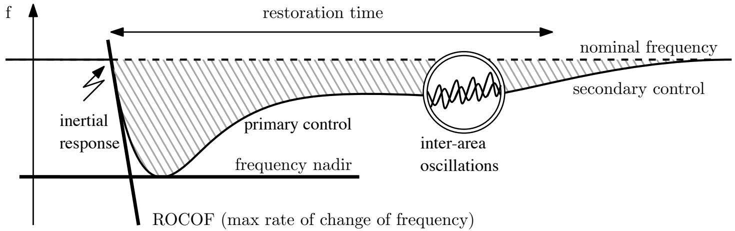

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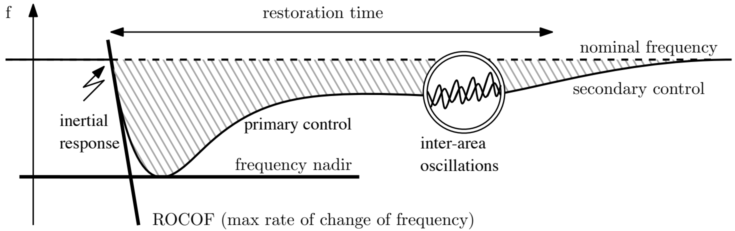

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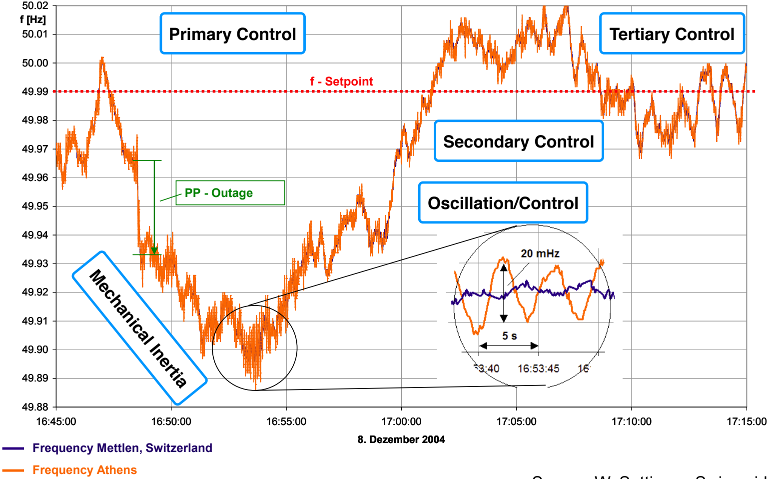
Response to disturbance in a **real** power system



Source: W. Sattinger, Swissgrid

When secondary frequency control goes wrong . . .

Grid projects About us Customers de fr it en

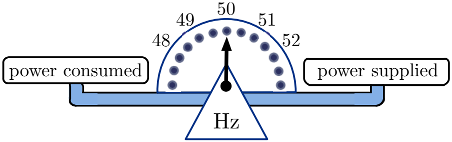
swissgrid

6 March 2018 | News

Frequency deviation in continental European grid leads to grid time deviations

"The missing energy amounts currently to 113 GWh . . . The decrease . . . is affecting also those electric clocks that are steered by the frequency of the power system . . . they show currently a delay of close to six minutes."

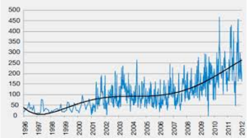
–ENTSO-E Press Release



Modern challenges for control engineers

1. Declining inertia and load-frequency responsiveness

   - **Sensitive** system
   - **Large** and **frequent** deviations



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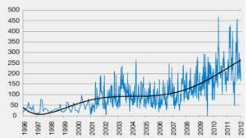

2. Heterogeneous small-scale power sources

   - Many **small** but **fast** sources
   - Fast freq. regulation markets

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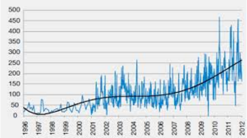

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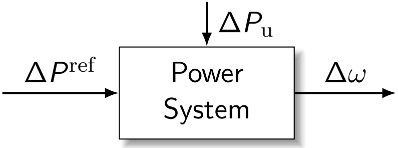
Opportunities for control engineers

1. Inverter-based resources and fast communication
2. Hierarchical control of **many** small devices

Simple dynamic models for frequency control (single area)



Control inputs:	Power set-points to devices $\Delta P_i^{\mathrm{ref}}$
Meas. / Controlled output:	Frequency error $\Delta\omega_i$
Unknown disturbances:	Uncontrolled load/generation $\Delta P_{\mathrm{u},i}$

$$
\begin{aligned}
\Delta\dot{\theta}_i &= \Delta\omega_i\,, \\
M_i \Delta\dot{\omega}_i &= -\sum\nolimits_{j=1}^{n} T_{ij}(\Delta\theta_i - \Delta\theta_j) - D_i\Delta\omega_i + \Delta P_{\mathrm{m},i} + \Delta P_{\mathrm{u},i} \\
T_i \Delta\dot{P}_{\mathrm{m},i} &= -\Delta P_{\mathrm{m},i} - R_{\mathrm{d},i}^{-1}\Delta\omega_i + \Delta P_i^{\mathrm{ref}}.
\end{aligned}
$$

(Note: Real governor model may be highly nonlinear!)

Fundamental insights from output regulation theory

Johnson, Davison, Francis, Wonham ...

System models are BIBO stable with steady-state given by

$$\Delta\omega = \underbrace{\beta^{-1}\mathbb{1}\mathbb{1}^{\mathsf{T}}}_{:=G_0 \text{ (DC Gain)}} (\Delta P_{\mathrm{u}} + \Delta P^{\mathrm{ref}}), \qquad \underbrace{\beta = \sum_i (D_i + R_{\mathrm{d},i}^{-1})}_{\text{Total proportional gain}}.$$

1. **Insight #1:** $\Delta\omega \in \mathrm{span}(\mathbb{1}_n) \iff$ "frequency is global"
2. **Insight #2:** Only *sum* of powers matters $\implies$ Resource allocation
3. **Insight #3:** $\mathrm{rank}(G_0) = 1$, which means ...

Only <u>one</u> frequency integrator permitted in <u>any</u> internally stable secondary control system!

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Optimal allocation of secondary resources

Optimally allocate inputs subject to **power balance** and **limits**

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} J_i(\Delta P_i^{\mathrm{ref}}) \\ \text{subject to} \quad & \sum_{i=1}^{n} \Delta P_i^{\mathrm{ref}} + \Delta P_{\mathrm{u},i} = 0 \\ & \Delta P_i^{\mathrm{ref}} \in \{\text{power limits}\} \,. \end{aligned}$$

Equivalent:

(i) Power balance

(ii) Regulation $\Delta\omega = 0$

Solve w/ $\Delta\omega$ feedback + Lagrange coordination

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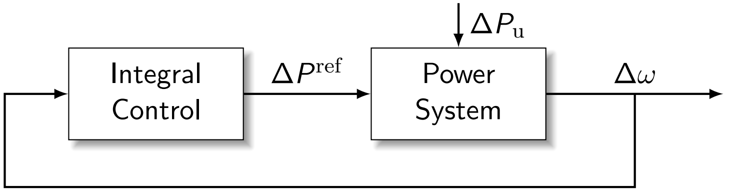

Hierarchical or distributed coordination

First-order **optimality condition**: $\exists\lambda$ s.t. $\forall i \quad \nabla J_i(\Delta P_i^{\mathrm{ref}}) = \lambda$.

How to enforce this **equal marginal cost** condition?

Hierarchical or distributed coordination

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How to enforce this **equal marginal cost** condition?

1. **Centralized** approach:

$$\underbrace{\tau\dot{\eta} = -\Delta\omega_{\mathrm{meas}}}_{\text{Central integral action}} \qquad \underbrace{\Delta P_i^{\mathrm{ref}} = (\nabla J_i)^{-1}(\eta)}_{\text{Allocation rule}}$$

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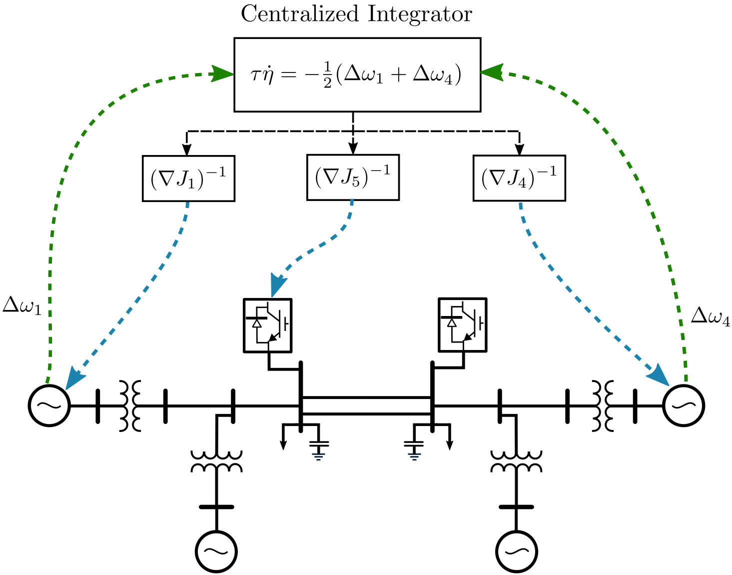
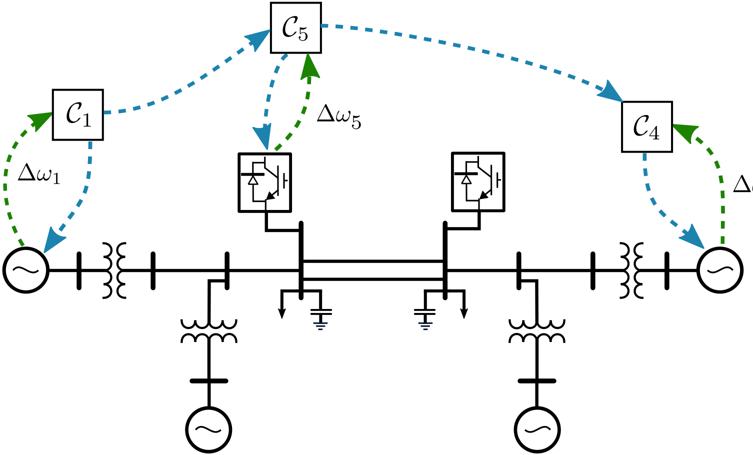
2. **Distributed** approach (consensus version):

$$\underbrace{\tau_i \dot{\eta}_i = -\Delta \omega_i - \sum_{j=1}^{n} a_{ij}(\eta_i - \eta_j)}_{\text{Distributed integral action}} \qquad \underbrace{\Delta P_i^{\mathrm{ref}} = (\nabla J_i)^{-1}(\eta_i)}_{\text{Allocation rule}}$$

Centralized architecture



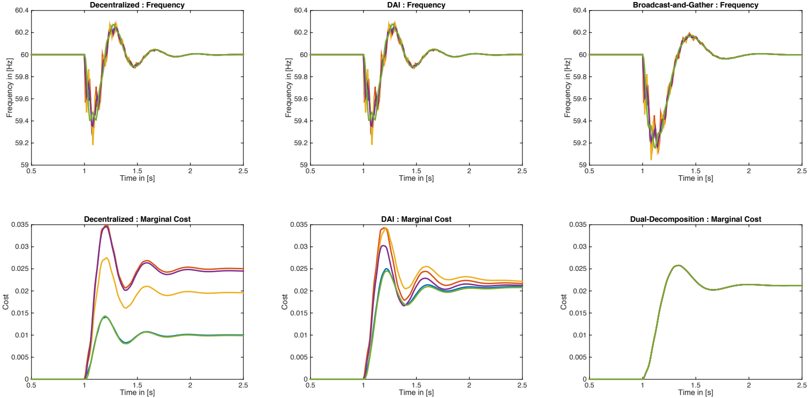
Distributed architecture



- Communication graph has a globally reachable node (nec. & suff.)

Distributed vs. centralized

Simulation on New England 39 Bus System[1]



[1] F. Dörfler and S. Grammatico, "Gather-and-broadcast frequency control in power systems," *Automatica*, 2017.

Hierarchical or distributed coordination contd.

3. **Distributed** approach (primal-dual version) uses
   - **virtual** phase angles $\hat{\theta}$
   - **virtual flow** on line $\ell = (i, j)$ as $\hat{p}_\ell = T_\ell(\hat{\theta}_i - \hat{\theta}_j)$

$$\begin{aligned} \text{minimize} \quad & \sum\nolimits_{i=1}^{n} J_i(\Delta P_i^{\mathrm{ref}}) + \tfrac{1}{2} D_i (\Delta\omega_i)^2 \\ \text{subject to} \quad & \Delta P_{\mathrm{u},i} + \Delta P_i^{\mathrm{ref}} - D_i \Delta\omega_i = \sum\nolimits_j A_{i\ell} \hat{p}_\ell \\ & \Delta P_{\mathrm{u},i} + \Delta P_i^{\mathrm{ref}} = \sum\nolimits_j T_{ij}(\hat{\theta}_i - \hat{\theta}_j) \end{aligned}$$

Now apply primal-descent and dual-ascent to **Lagrangian** $\mathcal{L}$

Primal dynamics: Embeds natural system dynamics

Dual dynamics: Distributed control

Hierarchical or distributed coordination contd.

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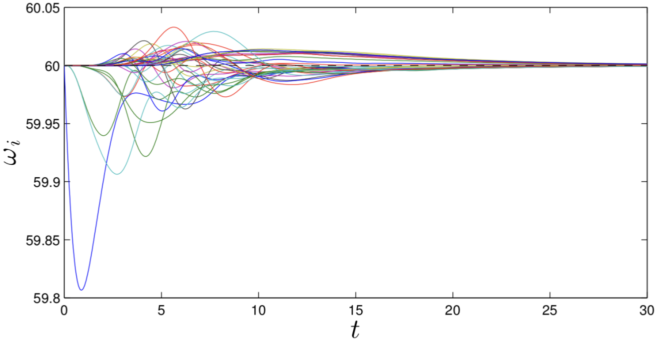
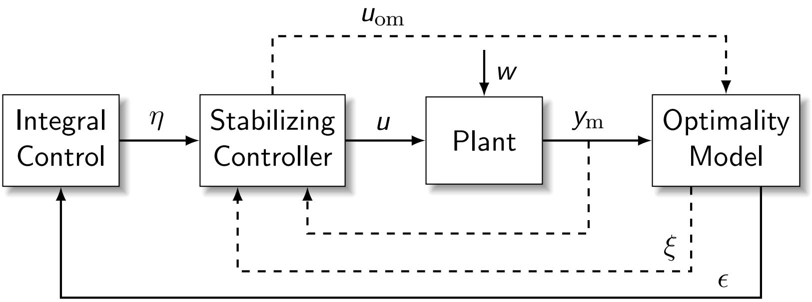
Primal-dual method

Simulation on New England 39 Bus System[2]



[2] E. Mallada and C. Zhao and S. Low, "Optimal Load-Side Control for Frequency Regulation . . . ," *IEEE TAC*, 2017.

Unifying perspective: Optimal Steady-State Control[3]



Optimality Model: creates optimality error signal ϵ

Integral Control: integrates error ϵ

Stabilizing Controller: stabilizes closed-loop system

[3] L. Lawrence, JWSP, E. Mallada "Linear-convex optimal steady-state control," *TAC*, Submitted.

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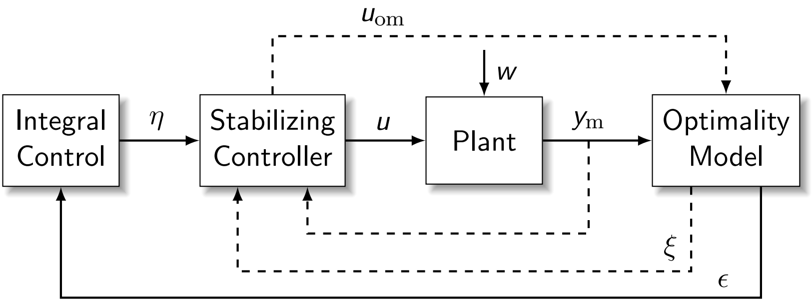

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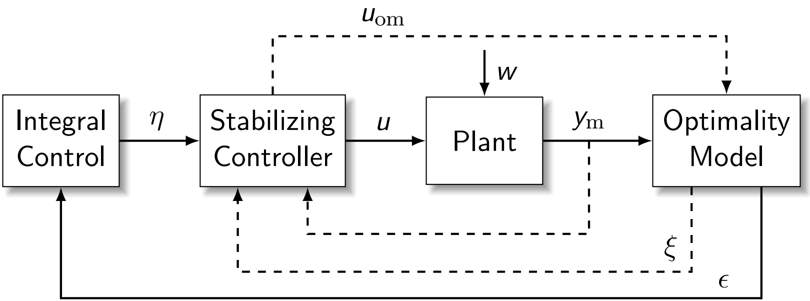

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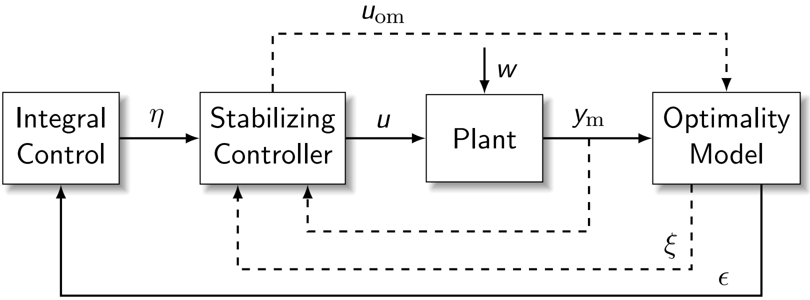

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Practical challenges

We have discussed **structural and architectural** aspects, but ...

1. Dynamic models are **highly** uncertain and time-varying
   - Device models often either unknown or not maintained
   - High-order governor models, deadbands, saturation all important
   - Machines dispatched in and out of system every $\approx$ 15 mins
   - Load characteristics change dramatically day-to-day
   - Even DC gain (β^{-1}) of system can vary by a factor of 2–3!

   Possible approach: **data-driven** + **gain-scheduled** methods

2. Communication infrastructure challenges
   - If it ain't broke, don't upgrade it
   - High-bandwidth control over comm. channels perceived as risky

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Questions

https://ece.uwaterloo.ca/~jwsimpso/
jwsimpson@uwaterloo.ca