On Stability of Automatic Generation Control

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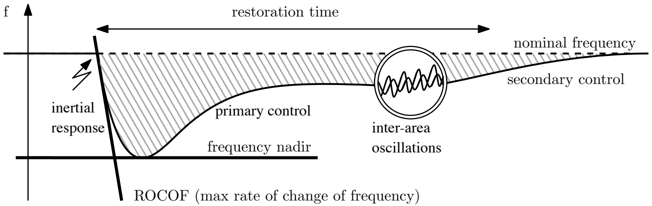
Frequency Control in Bulk Grid



Three stages of frequency control:

1. Inertial response: fast response of rotating machines
   Time scale: immediate/seconds
2. Primary control: turbine-governor control for *stabilization*
   Time scale: seconds
3. Automatic Generation Control (AGC): multi-area control which eliminates *generation-load mismatch* within each area
   Time scale: minutes

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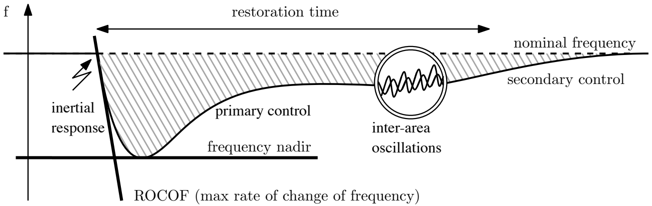

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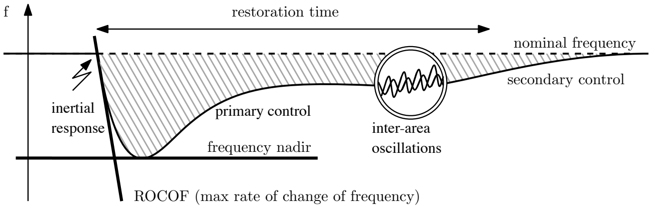

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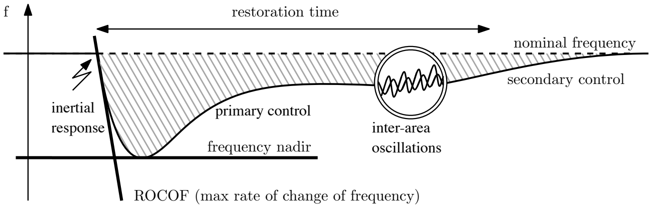

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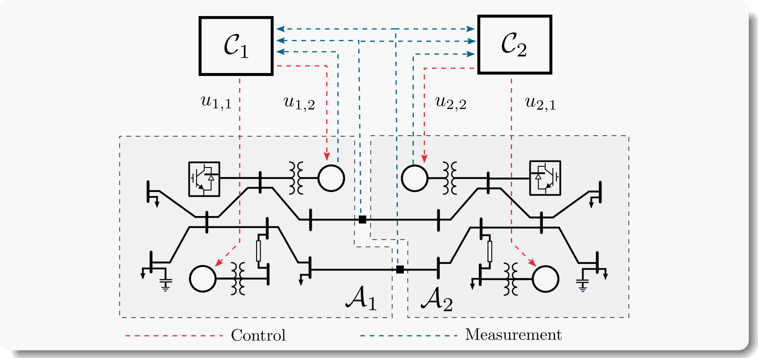
Automatic Generation Control



- interconnected system consisting of balancing authority areas
- decentralized integral control driven by area control error

$$\mathsf{ACE}_k(t) := \underbrace{\Delta \mathsf{NI}_k(t)}_{\text{Net Interchange}} + \underbrace{b_k \Delta f_k(t)}_{\text{Frequency Biasing}}$$

Automatic Generation Control

Characteristics:

- Area-by-area decentralized control, deployed since 1940's
- Eliminates generation-load mismatch within each area
- *AGC is slow compared to primary control dynamics*

Analysis:

- Textbook analysis considers only **equilibrium**
- 70+ years of research literature contains no formal dynamic analysis

Our Contribution: a definitive formal stability analysis of AGC in a fairly general interconnected nonlinear power system.

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Interconnected Power System Model

1. Interconnected system with **areas** $\mathcal{A} = \{1, \ldots, N\}$
2. $\mathcal{G}_k$ = set of **generators w/ turbine-gov systems** in area $k \in \mathcal{A}$
3. $\mathcal{G}_k^{\mathrm{AGC}} \subseteq \mathcal{G}_k$ = subset of gen which participate in AGC
4. power ref. to gen. $i \in \mathcal{G}_k^{\mathrm{AGC}} = u_{ki} \in [\underline{u}_{ki}, \overline{u}_{ki}]$, dispatch value $u_{k,i}^{\star}$
5. Δf_k = any frequency deviation measurement for area $k \in \mathcal{A}$
6. $\Delta \mathrm{NI}_k$ = net power flow (dev. from set-point) **out** of area $k \in \mathcal{A}$
7. Nonlinear interconnected power system model

$$\dot{x}(t) = F(x(t), u(t), w(t))$$
$$(\Delta f(t), \Delta \mathrm{NI}(t)) = h(x(t), u(t), w(t)),$$

where $w(t)$ = unmeasured disturbances and $u(t) \in \mathcal{U}$

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Technical Assumptions on Power System Model

There exist domains $\mathcal{X} \subseteq \mathbb{R}^n$ and $\mathcal{W} \subseteq \mathbb{R}^{n_w}$ such that the following hold:

1. **Model Regularity:** F, h, and Jacobians are Lipschitz cont. on $\mathcal{X}$ uniformly in $(u, w) \in \mathcal{U} \times \mathcal{W}$;
2. **Steady-State:** there exists a $\mathcal{C}^1$ map $x_{\rm ss} : \mathcal{U} \times \mathcal{W} \to \mathcal{X}$ which is Lipschitz on $\mathcal{U} \times \mathcal{W}$ and satisfies $0 = F(x_{\rm ss}(u, w), u, w)$ for all $(u, w) \in \mathcal{U} \times \mathcal{W}$;
3. **Stability:** the steady-state $x_{\rm ss}(u, w)$ is locally exponentially stable, uniformly in the inputs $(u, w) \in \mathcal{U} \times \mathcal{W}$;
4. **Steady-State Model:** the values $(\Delta f, \Delta \mathrm{NI}) = h(x_{\rm ss}(u, w), u, w)$ satisfy $\Delta f_1 = \Delta f_2 = \cdots = \Delta f_N$ and

$$0 = \sum_{k \in \mathcal{A}} \Delta \mathrm{NI}_k$$

$$\sum_{i \in \mathcal{G}_k} (P_{k,i} - u^\star_{k,i}) = D_k \Delta f_k + \Delta P^{\rm L}_k + \Delta \mathrm{NI}_k$$

$$P_{k,i} = u_{k,i} - \frac{1}{R_{k,i}} \Delta f_k$$

for each $k \in \mathcal{A}$ and $i \in \mathcal{G}_k$.

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Area Control Error and AGC Model

- **Recall:** ACE defined as

$$\mathsf{ACE}_k(t) := \Delta\mathsf{NI}_k(t) + b_k \Delta f_k(t)$$

- AGC controller for area k: **integrator** & **dispatch rule**

$$\begin{aligned} \tau_k \dot{\eta}_k(t) &= -\mathsf{ACE}_k(t) \\ u_{k,i} &= \mathrm{sat}_{k,i}(u^\star_{k,i} + \alpha_{k,i}\eta_k) \end{aligned}$$

- **time constants** $\tau_k \in [30s, 200\mathrm{s}]$
- constant **participation factors** $\alpha_{k,i}$ satisfy

$$\alpha_{k,i} \geq 0, \qquad \sum_{i \in \mathcal{G}_k^{\mathrm{AGC}}} \alpha_{k,i} = 1.$$

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Closed-Loop Stability under AGC

Main Theorem: Consider the interconnected power system with AGC under the previous assumptions. There exists $\tau^\star > 0$ such that if $\min_{k\in\mathcal{A}} \tau_k \geq \tau^\star$, then

1. the closed-loop system possesses a unique exponentially stable equilibrium point $(\bar{x}, \bar{\eta}) \in \mathcal{X} \times \mathbb{R}^N$, and
2. $\mathrm{ACE}_k(t) \to 0$ as $t \to \infty$ for all areas $k \in \mathcal{A}$.

Comments:

- result is **independent** of bias tunings $b_k > 0$
- consistent with **engineering practice**; no coordination required for stable tuning under usual time-scales of operation

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Proof Sketch

1. Set $\varepsilon = (\min_k \tau_k)^{-1}$ and let $t \mapsto \epsilon t$. Then CLS is

$$\begin{aligned} \varepsilon \dot{x} &= F(x,u,w) & \tilde{\tau}_k \dot{\eta}_k &= -(\Delta \mathrm{NI}_k + b_k \Delta f_k) \\ (\Delta f, \Delta \mathrm{NI}) &= h(x,u,w) & u_{k,i} &= \mathrm{sat}_{k,i}(u^\star_{k,i} + \alpha_{k,i}\eta_k), \end{aligned}$$

2. **Boundary layer dynamics** are uniformly exp. stable
3. Routine calculations to obtain vectorized **reduced dynamics**

$$\tilde{\tau}\dot{\eta} = \mathcal{B}(\varphi(\eta) - \Delta P^{\mathrm{L}})$$

where $\varphi_k(\eta_k) = \sum_{i \in \mathcal{G}_k} (\mathrm{sat}_{k,i}(u^\star_{k,i} + \alpha_{k,i}\eta_k) - u^\star_{k,i})$ and

$$\mathcal{B} := -\frac{1}{\beta}\begin{bmatrix} \beta + b_1 - \beta_1 & b_1 - \beta_1 & \cdots & b_1 - \beta_1 \\ b_2 - \beta_2 & \beta + b_2 - \beta_2 & \cdots & \cdots \\ \vdots & \cdots & \ddots & b_{N-1} - \beta_{N-1} \\ b_N - \beta_N & \cdots & b_N - \beta_N & \beta + b_N - \beta_N \end{bmatrix}.$$

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Lemma: The matrix $\mathcal{B}$ is diagonally stable, i.e., there exists a matrix $D = \mathrm{diag}(d_1, \ldots, d_N) \succ 0$ such that $\mathcal{B}^\mathsf{T} D + D\mathcal{B} \prec 0$.

5. Easy to argue that there exists unique $\bar{\eta}$ such that $\varphi(\bar{\eta}) = \Delta P^{\mathrm{L}}$, i.e., unique equilibrium $\bar{\eta}$ of the reduced dynamics

$$\tilde{\tau}\dot{\eta} = \mathcal{B}(\varphi(\eta) - \Delta P^{\mathrm{L}})$$

6. Lyapunov candidate $V : \mathbb{R}^N \to \mathbb{R}$ given by

$$V(\eta) = \sum_{k=1}^{N} d_k \tilde{\tau}_k \int_{\bar{\eta}_k}^{\eta_k} (\varphi_k(\xi_k) - \varphi_k(\bar{\eta}_k)) \, \mathrm{d}\xi_k.$$

establishes local exp. stability of $\bar{\eta}$ for reduced dynamics □

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6. Lyapunov candidate $V : \mathbb{R}^N \to \mathbb{R}$ given by

$$V(\eta) = \sum_{k=1}^{N} d_k \tilde{\tau}_k \int_{\bar{\eta}_k}^{\eta_k} (\varphi_k(\xi_k) - \varphi_k(\bar{\eta}_k)) \,\mathrm{d}\xi_k.$$

establishes local exp. stability of $\bar{\eta}$ for reduced dynamics □

Proof Sketch

Lemma: The matrix $\mathcal{B}$ is diagonally stable, i.e., there exists a matrix $D = \mathrm{diag}(d_1, \dots, d_N) \succ 0$ such that $\mathcal{B}^\mathsf{T} D + D\mathcal{B} \prec 0$.

5. Easy to argue that there exists unique $\bar{\eta}$ such that $\varphi(\bar{\eta}) = \Delta P^{\mathrm{L}}$, i.e., unique equilibrium $\bar{\eta}$ of the reduced dynamics

$$\tilde{\tau}\dot{\eta} = \mathcal{B}(\varphi(\eta) - \Delta P^{\mathrm{L}})$$

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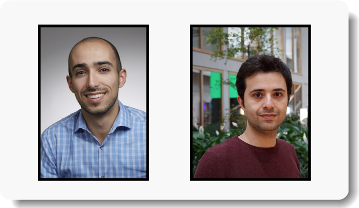
Conclusions

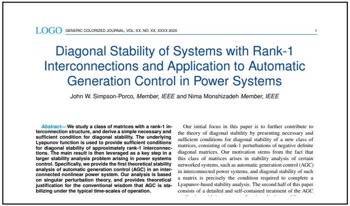
The first (to our knowledge) rigorous stability analysis of AGC

1. Singular perturbation theory, explicit Lyapunov construction
2. Theory backing 70 years of engineering practice

Future Work:

1. Incorporating governor deadband and network losses
2. Implications for tuning and modernizing AGC





LOGO GENERIC COLORIZED JOURNAL, VOL. XX, NO. XX, XXXX 2020 1

Diagonal Stability of Systems with Rank-1 Interconnections and Application to Automatic Generation Control in Power Systems

John W. Simpson-Porco, *Member, IEEE* and Nima Monshizadeh *Member, IEEE*

***Abstract*—We study a class of matrices with a rank-1 interconnection structure, and derive a simple necessary and sufficient condition for diagonal stability. The underlying Lyapunov function is used to provide sufficient conditions for diagonal stability of approximately rank-1 interconnections. The main result is then leveraged as a key step in a larger stability analysis problem arising in power systems control. Specifically, we provide the first theoretical stability analysis of automatic generation control (AGC) in an interconnected nonlinear power system. Our analysis is based on singular perturbation theory, and provides theoretical justification for the conventional wisdom that AGC is stabilizing under the typical time-scales of operation.**

Our initial focus in this paper is to further contribute to the theory of diagonal stability by presenting necessary and sufficient conditions for diagonal stability of a new class of matrices, consisting of rank-1 perturbations of negative definite diagonal matrices. Our motivation stems from the fact that this class of matrices arises in stability analysis of certain networked systems, such as automatic generation control (AGC) in interconnected power systems, and diagonal stability of such a matrix is precisely the condition required to complete a Lyapunov-based stability analysis. The second half of this paper consists of a detailed and self-contained treatment of the AGC

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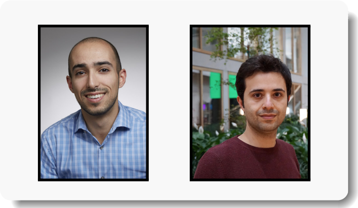

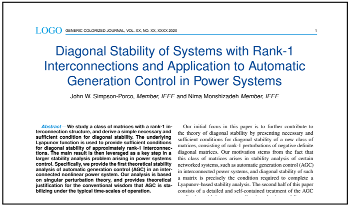

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Questions

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