The Return of the Tuning Regulator

John W. Simpson-Porco

https://www.control.utoronto.ca/~jwsimpson/

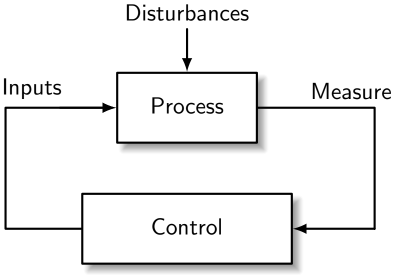
The Edward S. Rogers Sr. Department
of Electrical & Computer Engineering
UNIVERSITY OF TORONTO

9th Meeting on System and Control Theory

University of Waterloo

May 4, 2023

Introduction



- Process complex / nonlinear / unknown but **stable** and **responsive to inputs**
- Design criteria
  - (i) want to track and reject
  - (ii) use minimal model info.
  - (iii) robustness $\gg$ performance

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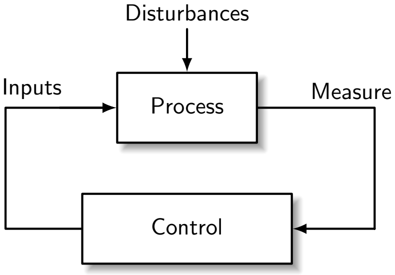

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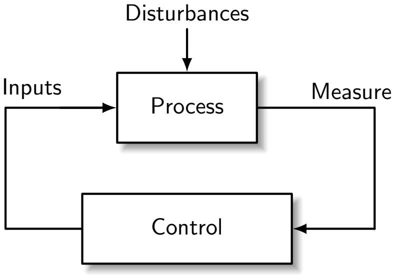

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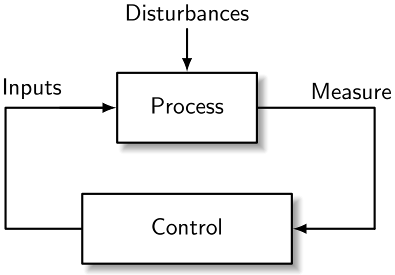

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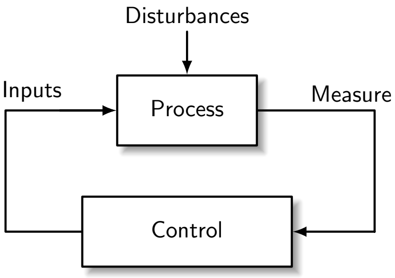

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Problem falls within the well-studied topic of **output regulation** or **servomechanism design**

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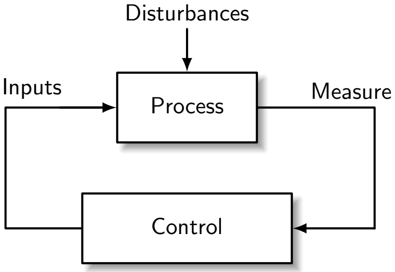

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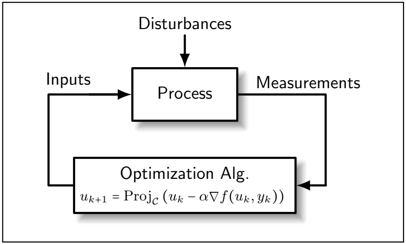
Objective: Design low-order, easily tuned, and (nearly) model-free regulation loops for stable MIMO systems

My Motivating Applications

Feedback-Based Optimization

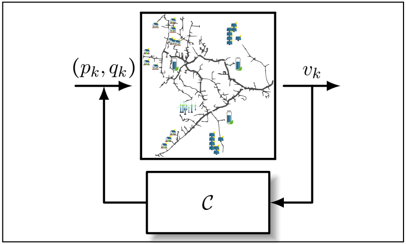


Analysis of Optimization Algorithms

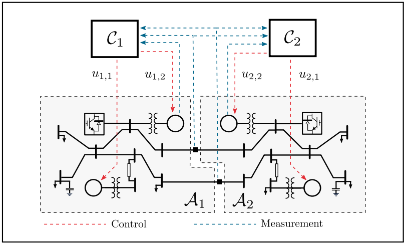
$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \; f(x) \quad \text{s.t.} \quad Ax = b$$

$$\begin{aligned} x_{k+1} &= x_k - \alpha\left(\nabla f(x_k) + A^{\mathsf{T}} \lambda_k\right) \\ \tilde{x}_k &= x_k + \gamma\left(x_{k+1} - x_k\right) \\ \lambda_{k+1} &= \lambda_k + \beta\left(A\tilde{x}_k - b\right), \end{aligned}$$

Renewable Energy Integration



Next-Generation Grid Control



This Talk

4148 IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 66, NO. 9, SEPTEMBER 2021

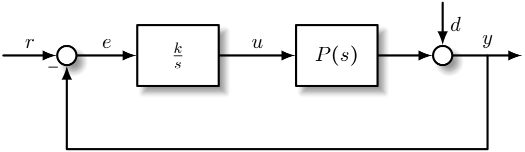
Analysis and Synthesis of Low-Gain Integral Controllers for Nonlinear Systems

John W. Simpson-Porco, *Member, IEEE*

Data-Driven Output Regulation using Single-Gain Tuning Regulators

Liangjie Chen and John W. Simpson-Porco

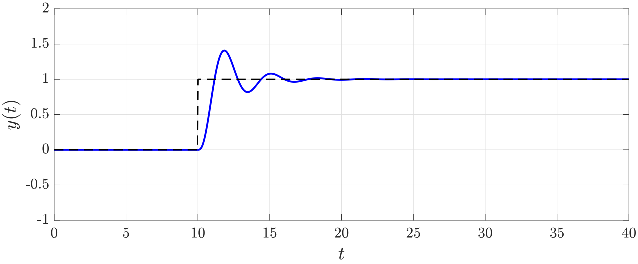
Fundamentals of Integral Control



Assume P stable, $P(0) \neq 0$. What are the basic facts and tuning principles?

The Integral Control Dichotomy: Either

(a) the closed-loop is BIBO stable **and** $\lim_{t\to\infty} e(t) = 0$, or



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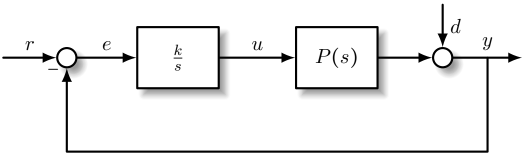

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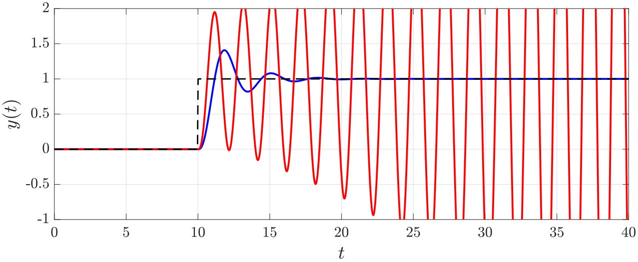

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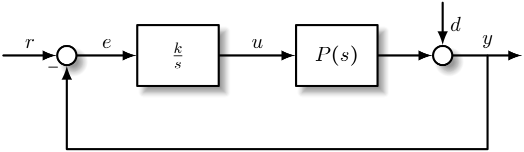

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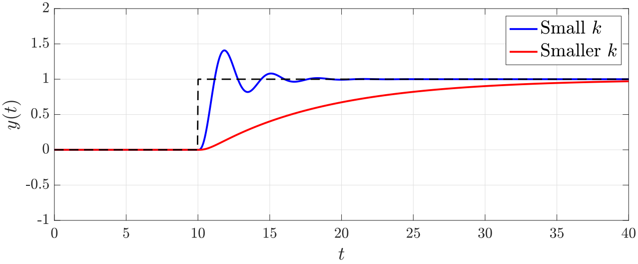
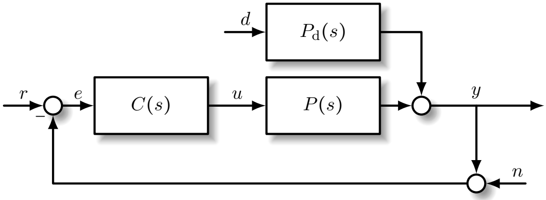


Low-Gain Integral Control of LTI Systems



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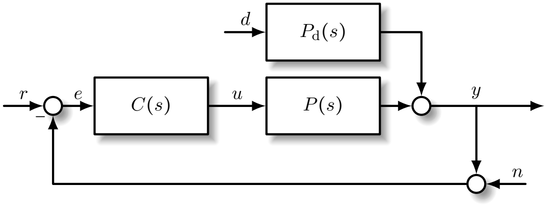

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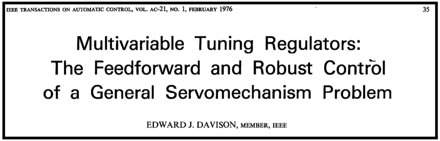

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. AC-21, NO. 1, FEBRUARY 1976 35

Multivariable Tuning Regulators: The Feedforward and Robust Control of a General Servomechanism Problem

EDWARD J. DAVISON, MEMBER, IEEE

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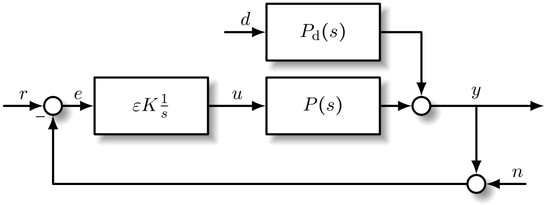

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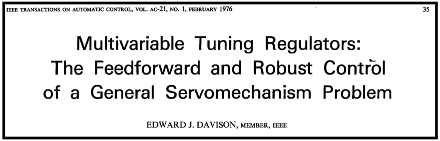
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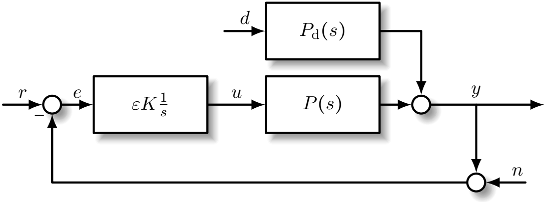

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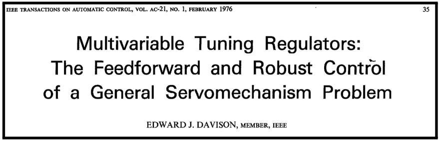

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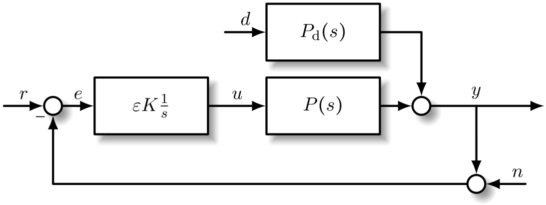

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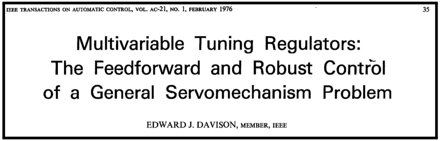

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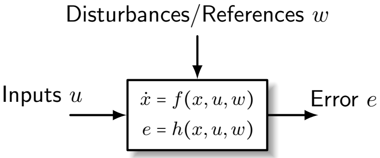
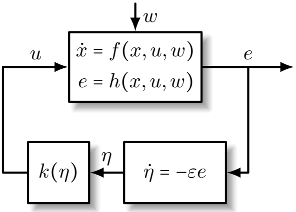
Only required model information is the DC gain!

Low-Gain Integral Control of Nonlinear Systems





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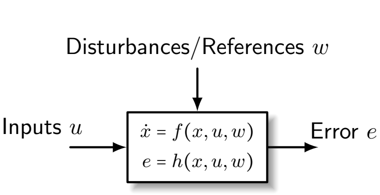

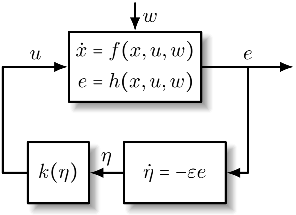

Assumptions: There exist state and input sets $\mathcal{X}$ and $\mathcal{I}$ such that

1. **Model Regularity:** f and $\frac{\partial f}{\partial x}$ are Lipshitz on $\mathcal{X}$ uniformly in $(u, w) \in \mathcal{I}$,
2. **Steady-State:** $\exists\ \pi_x : \mathcal{I} \to \mathcal{X}$ s.t. $0 = f(\pi_x(u, w), u, w)$ for all $(u, w) \in \mathcal{I}$,
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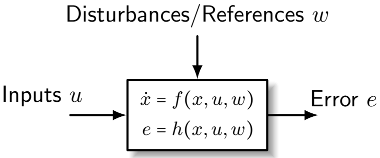

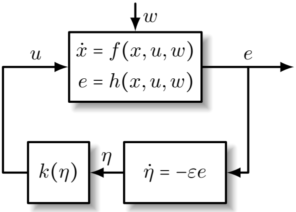

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Equilibrium I/O Map:

$$\pi(\bar{u}, w) \triangleq h(\pi_x(\bar{u}, w), \bar{u}, w)$$
$$= P(0)\bar{u} + P_w(0)w \text{ for LTI}$$

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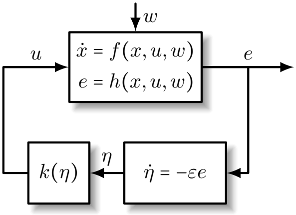

Equivalent

1. ε is **small**
2. integral action is **slow**
3. process is **fast**

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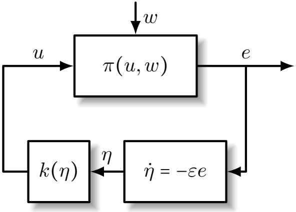

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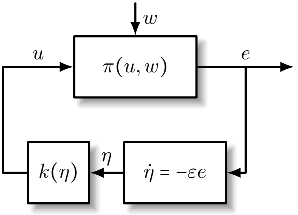

Reduced Dynamics

$$\frac{1}{\varepsilon}\dot{\eta} = -\pi(k(\eta), w) \triangleq F_w(\eta)$$
$$= -P(0)K\eta - P_w(0)w$$

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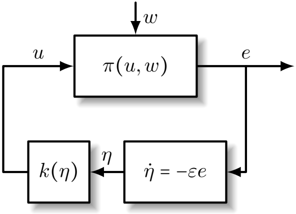

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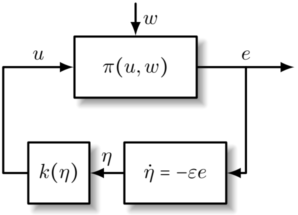

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Contraction is an incremental stability concept which furnishes nonlinear systems $\dot{x} = f(x)$ with linear-like stability properties.

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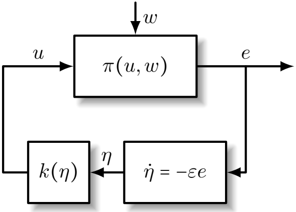
Suppose $f \in \mathsf{C}^1$ is globally **infinitesimally contracting**, i.e.,

$$\sup_{x\in\mathbb{R}^n} \mu\left(\frac{\partial f}{\partial x}(x)\right) < 0.$$

Then $\dot{x} = f(x)$ has a **unique globally exponentially stable** equilibrium point $\bar{x}$.

Low-Gain Integral Control of Nonlinear Systems



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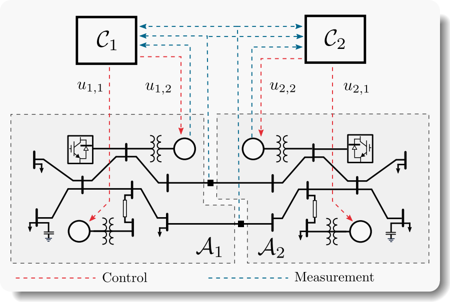
Application: Automatic Generation Control

Rebalancing supply and demand in interconnected systems



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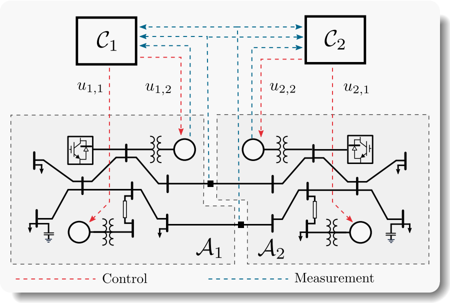

- Area Control Error (ACE)

$$\mathsf{ACE}_k(t) := \underbrace{\Delta \mathsf{NI}_k(t)}_{\text{Net Interchange}} + \underbrace{b_k \Delta f_k(t)}_{\text{Frequency Biasing}}$$

- Area-based integral controller

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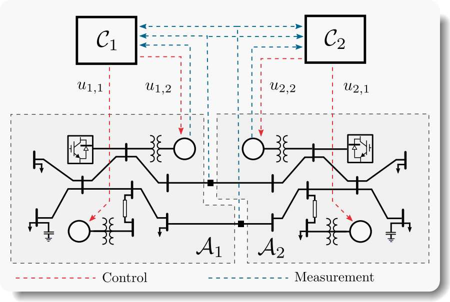

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- Decentralized; eliminates *generation-load mismatch*

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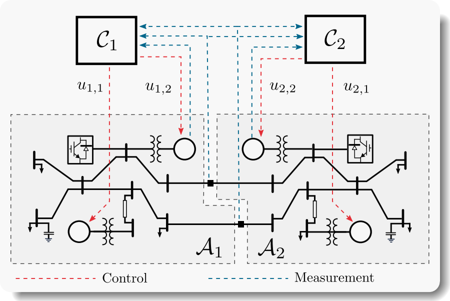

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$$\tau_k \dot{\eta}_k = -\mathsf{ACE}_k$$
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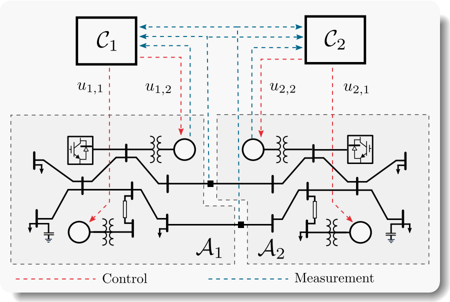
- Decentralized; eliminates *generation-load mismatch*
- Operating continuously in North America since about 1950; 70+ years of research literature contained no formal dynamic analysis

Application: Automatic Generation Control

Rebalancing supply and demand in interconnected systems



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- Definitive analysis enabled by previous result, and some advances in theory of diagonal stability; see paper in Trans. Control Network Systems

Designing Low-Gain Integral Controllers

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(ii) **Robust Design**: If you can represent/encapsulate $\pi(u, w)$ in a linear fractional model

$$\begin{aligned} e &= Fu + G\mathsf{p} + E_1 w \\ \mathsf{q} &= Hu + J\mathsf{p} + E_2 w \end{aligned} \qquad \mathsf{p} = \Delta(\mathsf{q})$$

where Δ satisfies **incremental quadratic constraints**; can immediately apply SDP-based robust/optimal design methods

Davison's Tuning Regulator

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. AC-21, NO. 1, FEBRUARY 1976 35

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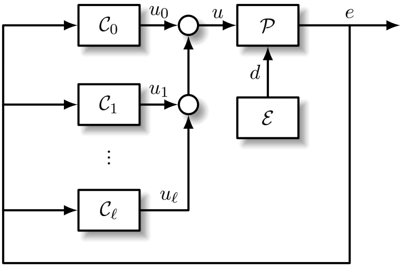
For Simplicity: $\mathrm{MinPoly}_S(s) = s(s^2+\omega_1^2)\cdots(s^2+\omega_\ell^2)$

Necessarily: $\mathrm{rank}\,\hat{P}(\lambda) = r$ for all $\lambda \in \mathrm{eig}(S)$.

Architecture of Davison's Tuning Regulator

Idea: Sub-controllers handle each disturbance mode



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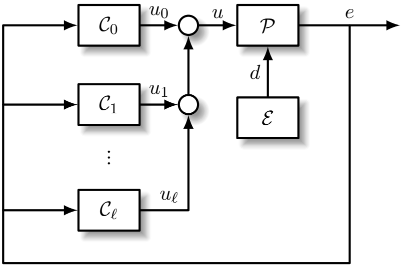

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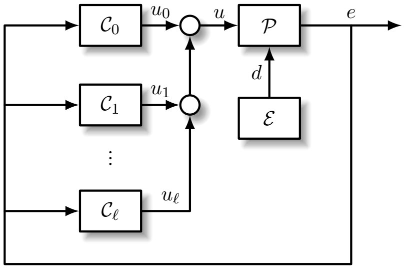

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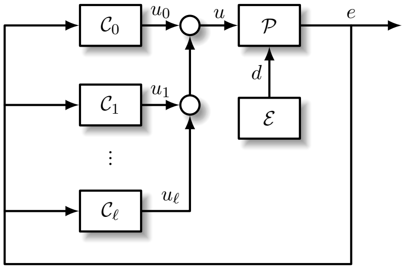

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- **Want:** Adjusting ϵ adjusts dominant closed-loop poles
- **Definition:** $\mathcal{A}(\epsilon)$ is *low-gain stable* if there exist $c, \epsilon^\star > 0$ such that $\mathrm{Re}(\lambda) \leq -c\epsilon$ for all $\lambda \in \mathrm{eig}(\mathcal{A}(\epsilon))$ and all $\epsilon \in [0, \epsilon^\star)$.

Reduction of Stability Analysis Problem

$$\mathcal{A}(\epsilon) := \begin{bmatrix} A & -BF(\epsilon) \\ GC & \Phi - GDF(\epsilon) \end{bmatrix}$$

is low-gain stable

$$\iff$$

$$\mathcal{A}_{\text{red}}(\epsilon) = \Phi - G\mathscr{L}(F(\epsilon))$$

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where $\mathscr{L} : \mathbb{R}^{m \times rq} \to \mathbb{R}^{r \times rq}$

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Idea: Place poles of (Φ, G) with feedback gain K(ϵ), then solve linear operator equation $\mathscr{L}(F)$ = K = for F.

The Operator Equation $\mathscr{L}(F) = \mathsf{K}$

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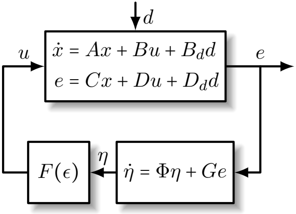
$$\mathscr{L}(F) = \hat{P}(0)F\boldsymbol{X}_0 + 2\sum_{k=1}^{\ell} \mathrm{Re}\{\hat{P}(\mathbf{j}\omega_k)F\boldsymbol{X}_k\}.$$

where matrix $\boldsymbol{X}_k$ comes from eigen. decomp. of Φ.

Summary



Frequency response data $\hat{P}(0)$ and $\hat{P}(\mathbf{j}\omega_k)$ can be inferred from measurements; **full plant model irrelevant**

Design Procedure:

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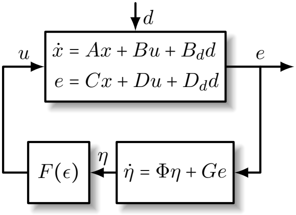

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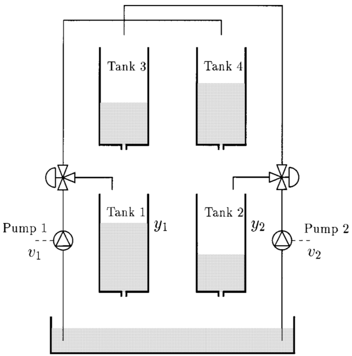
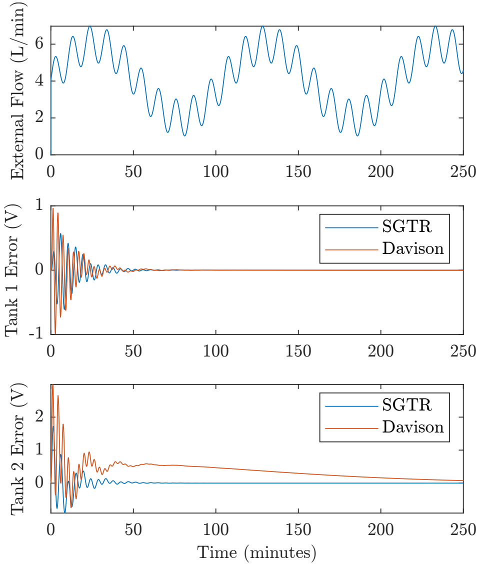
Full design authority over slow time-scale dynamics.

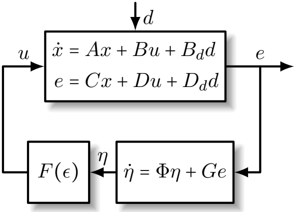
Example: Four-Tank Process (Johannson, TCST, 2000)





Conclusions

1. Low-complexity easily-tuned robust data-driven design
2. Integral controllers for nonlinear systems
3. Improvement to tuning regulators for LTI systems



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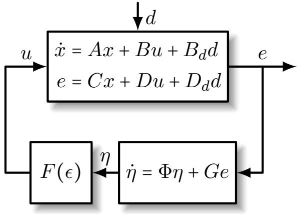

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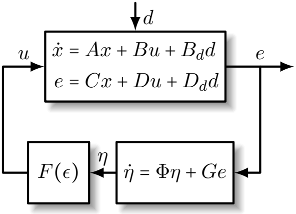

Note: Open PhD position at University of Toronto for Fall 2023, focusing on data-driven control and estimation for energy systems.

Questions

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