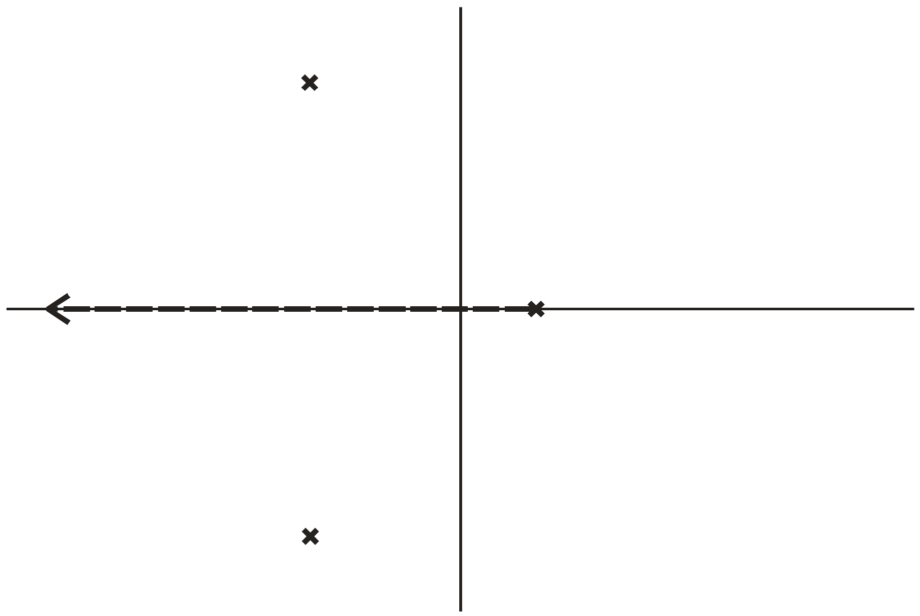
Roughly sketch the root locus for the pole zero maps figure 5.57e and 5.57f including

- Loci on the real axis
- Asymptotes & Centroid
- rough Arrival & Departure angles

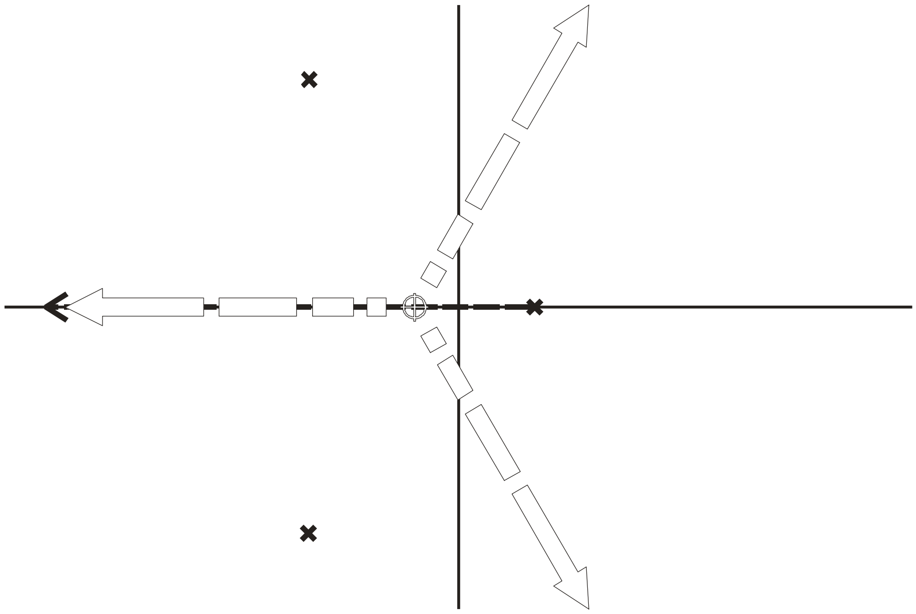
5.2e) First estimate the pole and zero locations for use in rough calculations. There are 3 poles at approximately $-2+3i$, $-2-3i$, 1

Now the loci on the real axis. The rule is if the #poles + #zeros to the right of a point on the real axis is odd then that point is on the locus.



Next fill in the asymptotic locus, including centroid α, and asymptotes with angles ϕ_l.

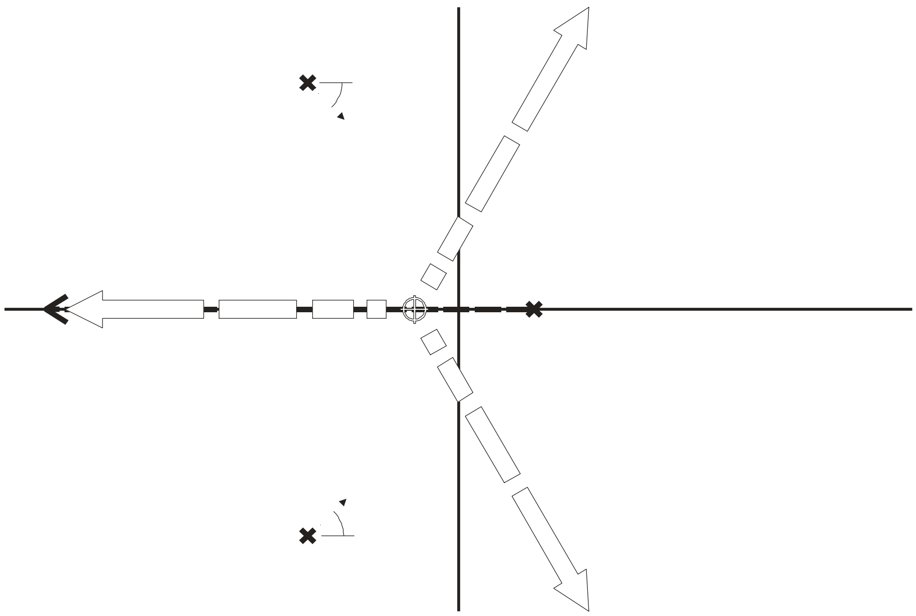
$$
\begin{aligned}
n-m &= \textit{number of asymptotes} \\
&= 3 \\
\alpha &= \frac{\sum p_i - \sum z_i}{n-m} \\
&\approx \frac{(-2-3i)+(-2+3i)+(1)}{3} \\
&= -1 \\
\phi_l &= \frac{180^\circ + 360^\circ(l-1)}{n-m}, \qquad l = 1,2,\ldots,(n-m) \\
&= \frac{180^\circ}{3}, \ \frac{180^\circ + 360^\circ}{3}, \ \frac{180^\circ + 720^\circ}{3} \\
&= 60^\circ, \ 180^\circ, \ 300^\circ
\end{aligned}
$$



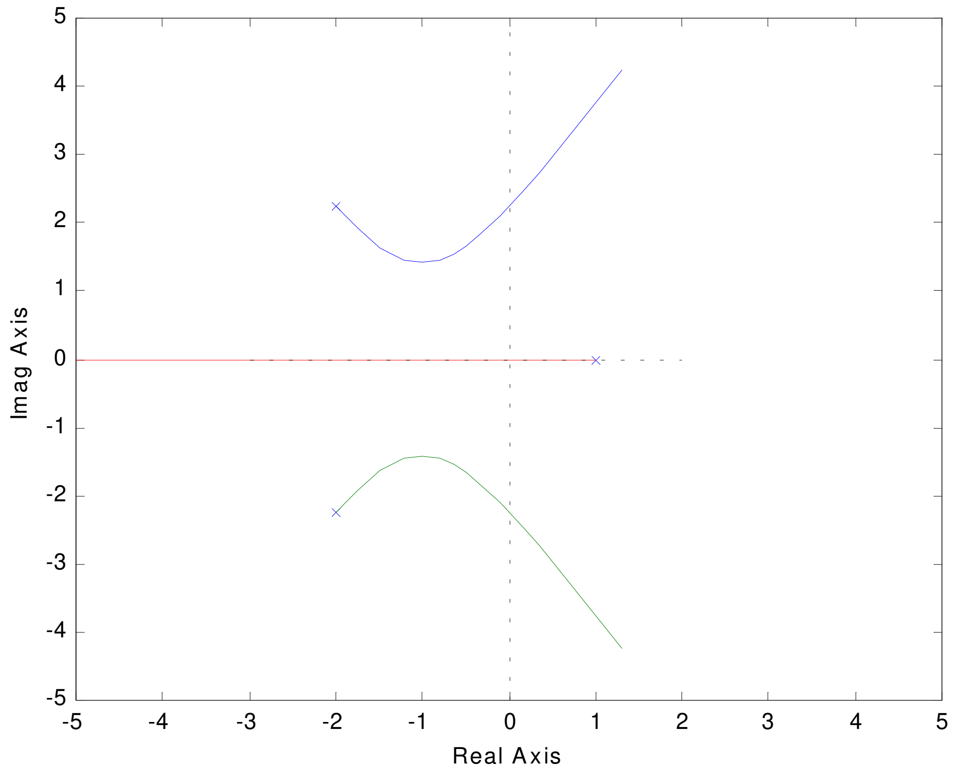
Finally estimate departure angles. The pole at $1+0i$ will depart at 180°. From the pole located at $-2+3i$ the departure angle is calculated as follows:

$$\begin{aligned}\phi_{dep} &= \sum \psi_i - \sum \phi_i - 180^{\circ} - 360^{\circ} \\ &= -90^{\circ} - \left(\tan^{-1}\left(\frac{3}{3}\right) + 90^{\circ} \right) - 180^{\circ} \\ &= -405 \; or \; -45^{\circ}\end{aligned}$$

And by symmetry the departure angle from the pole at $-2-3i$ is 45°

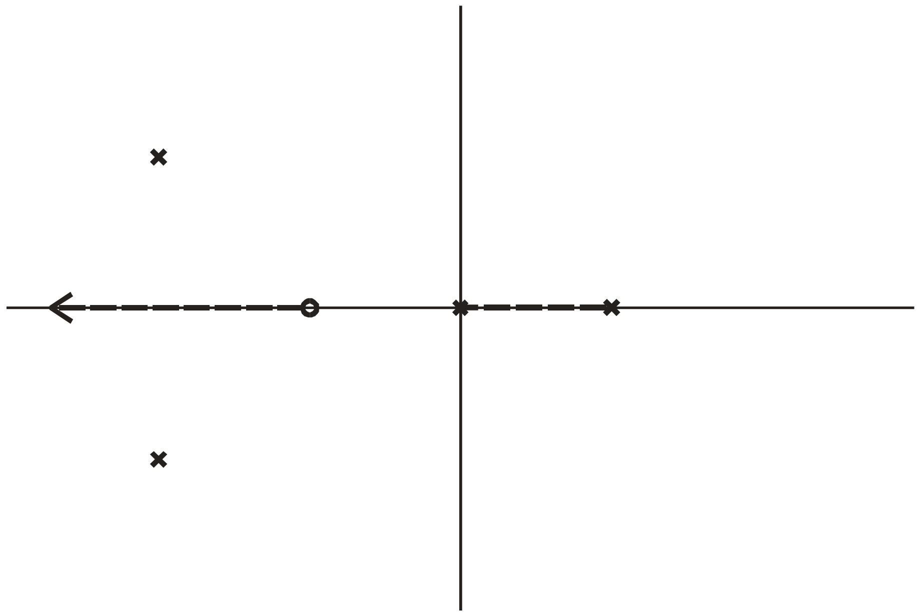


Finally fill in the locus, here is a MATLAB plot.



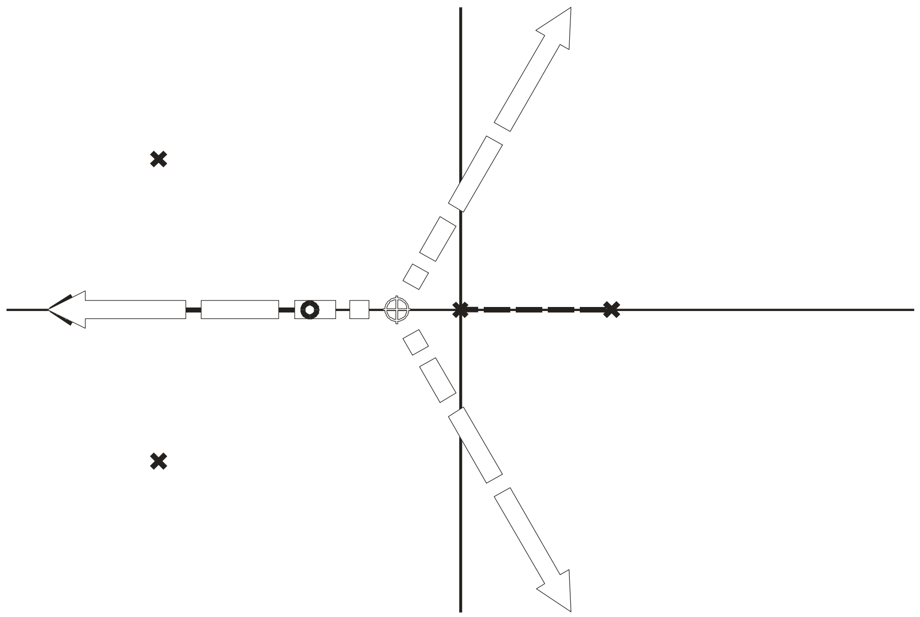
5.2f) First estimate the pole and zero locations for use in rough calculations. There are 4 poles at approximately $-2+i$, $-2-i$, 0, 1 and there is one zero at –1.

Now the loci on the real axis. The rule is if the #poles + #zeros to the right of a point on the real axis is odd then that point is on the locus.



Next fill in the asymptotic locus, including centroid α, and asymptotes with angles ϕ_l.

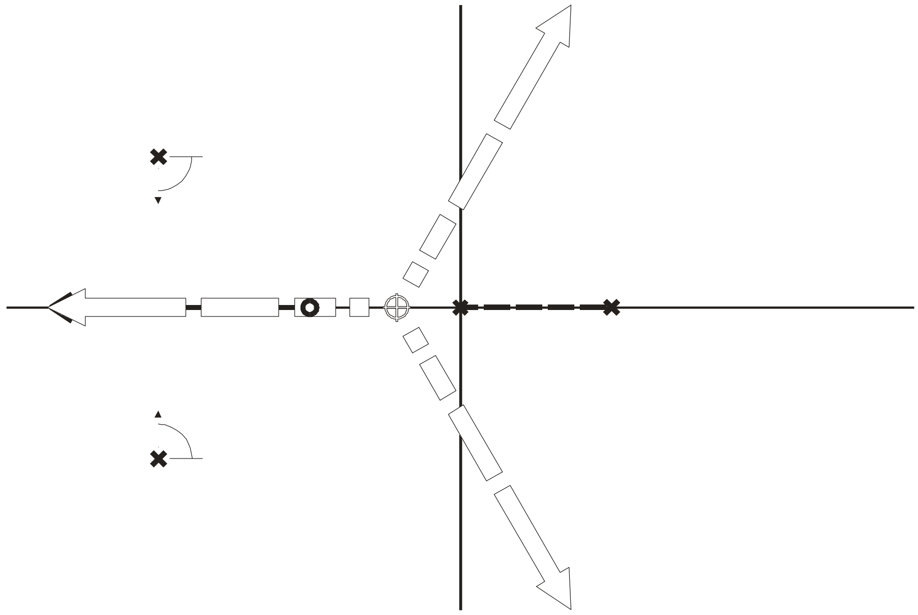
$$
\begin{aligned}
n-m &= \textit{number of asymptotes} \\
&= 3 \\
\alpha &= \frac{\sum p_i - \sum z_i}{n-m} \\
&\approx \frac{(-2-i)+(-2+i)+(1)+(0)-(-1)}{3} \\
&= -2/3 \\
\phi_l &= \frac{180^\circ + 360^\circ (l-1)}{n-m}, \qquad l = 1,2,\ldots,(n-m) \\
&= \frac{180^\circ}{3}, \ \frac{180^\circ + 360^\circ}{3}, \ \frac{180^\circ + 720^\circ}{3} \\
&= 60^\circ,\ 180^\circ,\ 300^\circ
\end{aligned}
$$



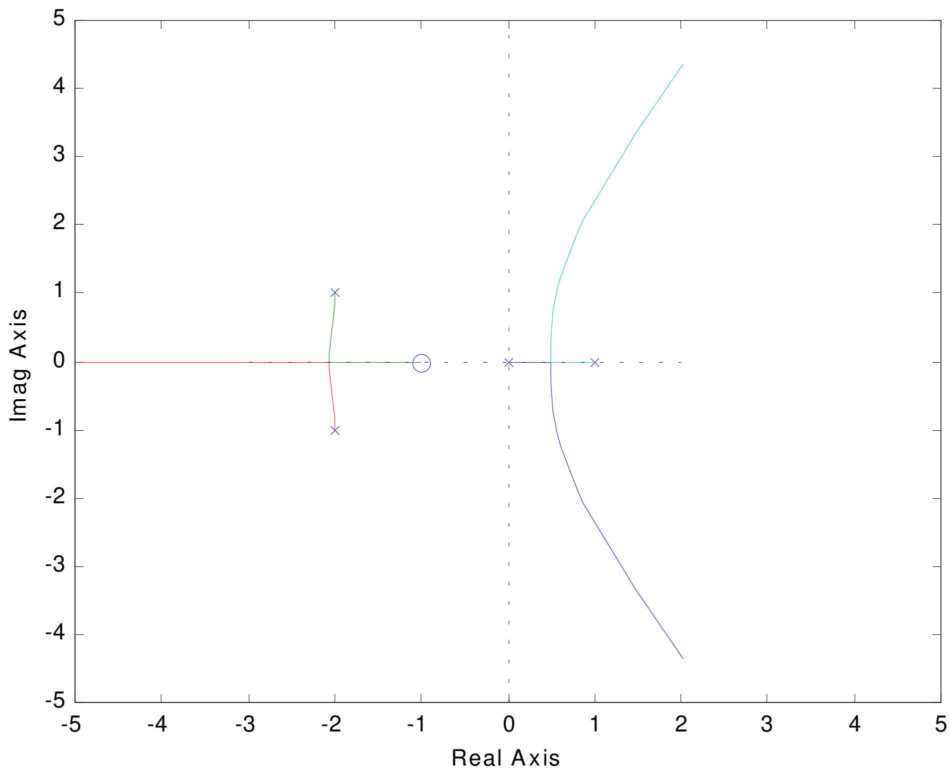
Finally estimate arrival & departure angles. By inspection it can be seen that the poles at $0+0i$, and $1+0i$ will depart at 0° and 180° respectively. From the pole located at $-2+i$ the departure angle is calculated as follows:

$$
\begin{aligned}
\phi_{dep} &= \sum \psi_i - \sum \phi_i - 180^\circ - 360^\circ \\
&= \left(\tan^{-1}\left(\frac{1}{1}\right) + 90^\circ \right) - 90^\circ - \left(\tan^{-1}\left(\frac{2}{1}\right) + 90^\circ \right) - \left(\tan^{-1}\left(\frac{3}{1}\right) + 90^\circ \right) - 180^\circ \\
&= 135^\circ - 90^\circ - 153^\circ - 162^\circ - 180^\circ = -450^\circ \ \textit{or} \ -90^\circ
\end{aligned}
$$

And by symmetry the departure angle from the pole at $-2-i$ is 90°. By inspection it is clear that the arrival angle at the zero will be 180°



And finally fill in the locus.

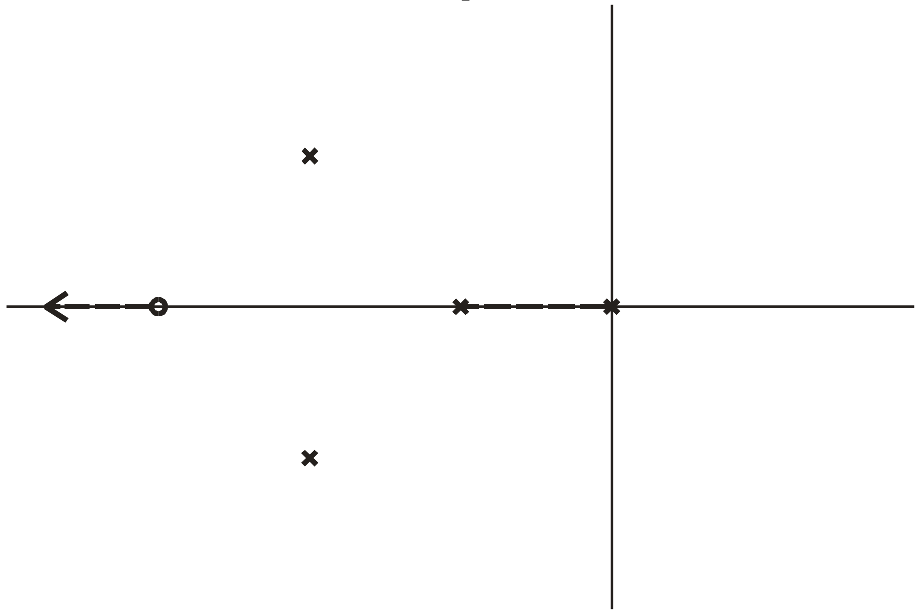


Sketch the root locus for 5.4d and 5.5f including

- Loci on the real axis
- Asymptotes & centroid
- Arrival & departure angles
- Imaginary axis crossings
- MATLAB verification

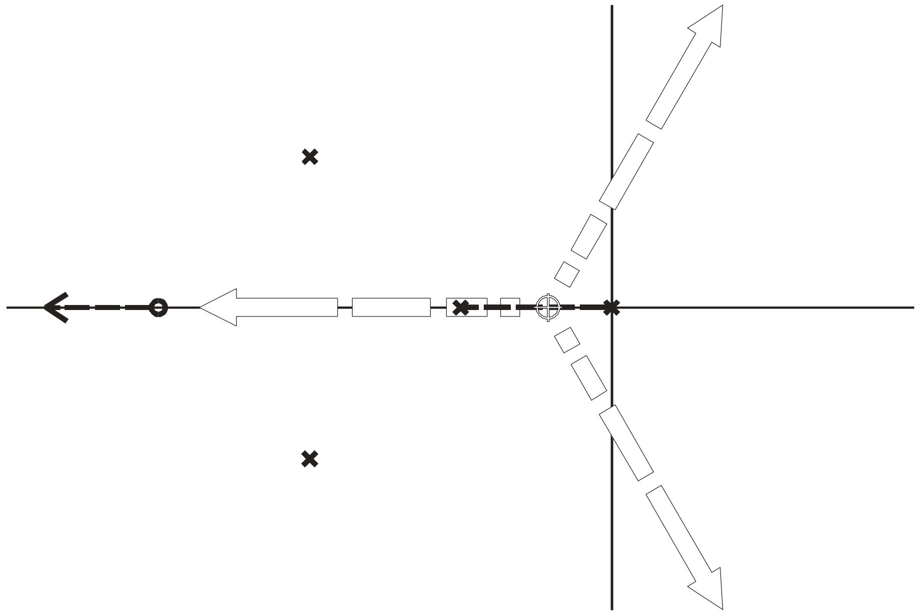
5.4d) First determine the pole and zero locations. There are 4 poles at 0, -1, $-2+i$, and $-2-i$ respectively, and there is one zero at -3.

Now the loci on the real axis. The rule is if the #poles + #zeros to the right of a point on the real axis is odd then that point is on the locus.



Next fill in the asymptotic locus, including centroid α, and asymptotes with angles ϕ_l.

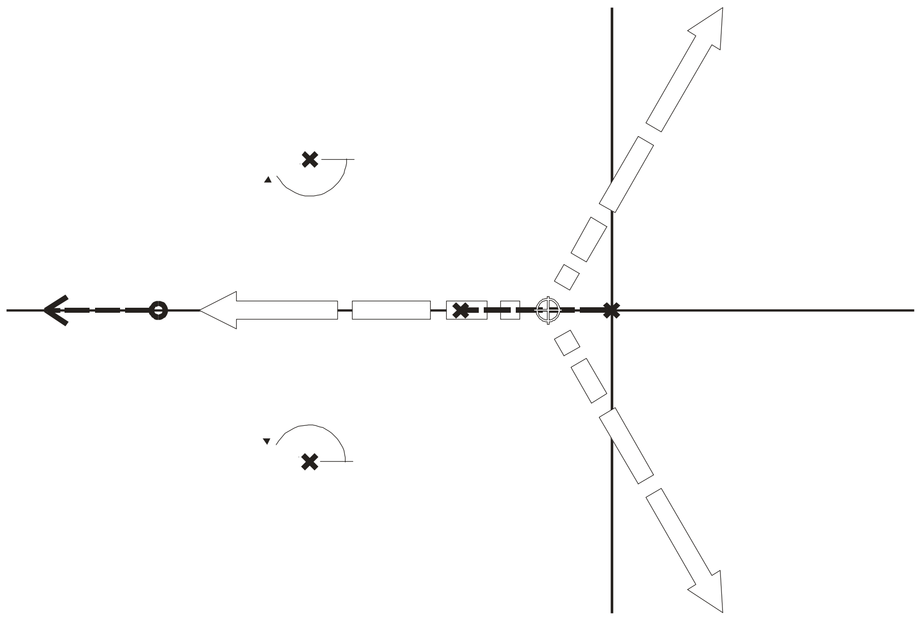
$$
\begin{aligned}
n-m &= \textit{number of asymptotes}\\
&= 3\\
\alpha &= \frac{\sum p_i - \sum z_i}{n-m}\\
&\approx \frac{(-2-i)+(-2+i)+(-1)+(0)-(-3)}{3}\\
&= -2/3\\
\phi_l &= \frac{180^\circ + 360^\circ(l-1)}{n-m}, \qquad l = 1,2,\ldots,(n-m)\\
&= \frac{180^\circ}{3}, \ \frac{180^\circ+360^\circ}{3}, \ \frac{180^\circ+720^\circ}{3}\\
&= 60^\circ, \ 180^\circ, \ 300^\circ
\end{aligned}
$$



Finally estimate arrival & departure angles. By inspection it can be seen that the poles at $-1+0i$, and $0+0i$ will depart at 0° and 180° respectively. From the pole located at $-2+i$ the departure angle is calculated as follows:

$$\begin{aligned}\phi_{dep} &= \sum \psi_i - \sum \phi_i - 180^{\circ} - 360^{\circ} \\ &= \tan^{-1}\left(\frac{1}{1}\right) - 90^{\circ} - \left(\tan^{-1}\left(\frac{1}{1}\right) + 90^{\circ}\right) - \left(\tan^{-1}\left(\frac{2}{1}\right) + 90^{\circ}\right) - 180^{\circ} \\ &= 45^{\circ} - 90^{\circ} - 135^{\circ} - 153^{\circ} - 180^{\circ} = -513^{\circ} \; or \; -153^{\circ}\end{aligned}$$

And by symmetry the departure angle from the pole at $-2-i$ is 153°. By inspection it is clear that the arrival angle at the zero will be 180°.



To find the imaginary axis crossings solve for roots of the characteristic polynomial (or closed loop denominator) when $s = j\omega$.

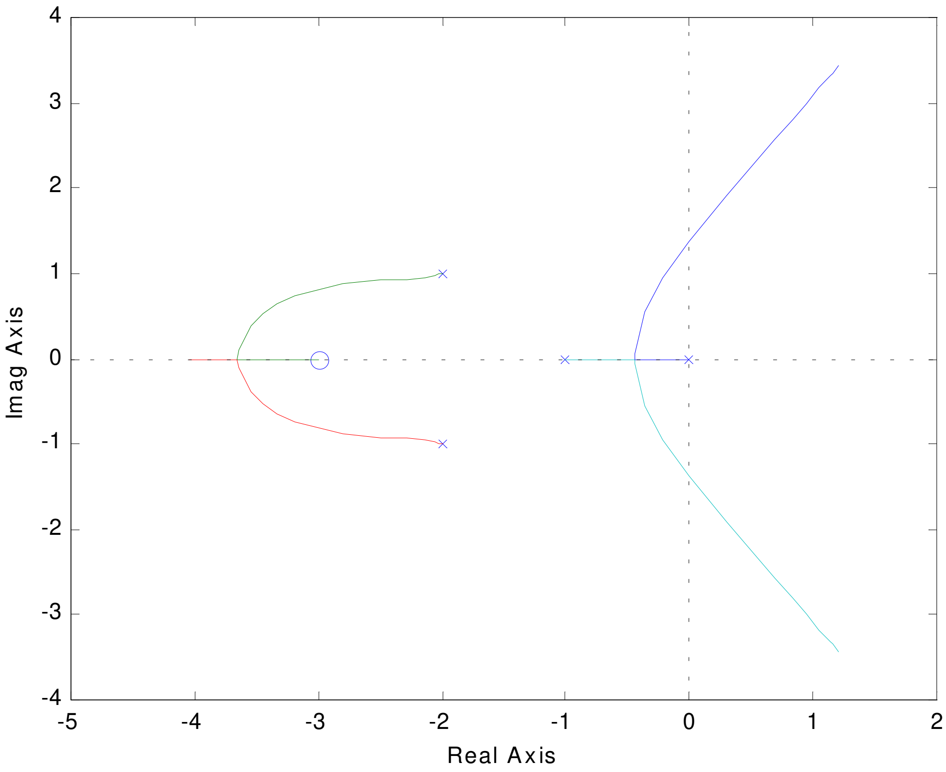
$$0 = 1 + KG(s)$$
$$0 = s^4 + 5s^3 + 9s^2 + 5s + Ks + 3K$$
$$s = j\omega$$
$$0 = \omega^4 - j5\omega^3 - 9\omega^2 + j\omega(5 + K) + 3K$$

Re	Im
$0 = \omega^4 - 9\omega^2 + 3K$	$0 = -j5\omega^3 + j\omega(5 + K)$
	$K = 5\omega^2 - 5$

$$0 = \omega^4 + 6\omega^2 - 15$$
$$\omega^2 = \frac{-6 \pm \sqrt{36 + 60}}{2}$$

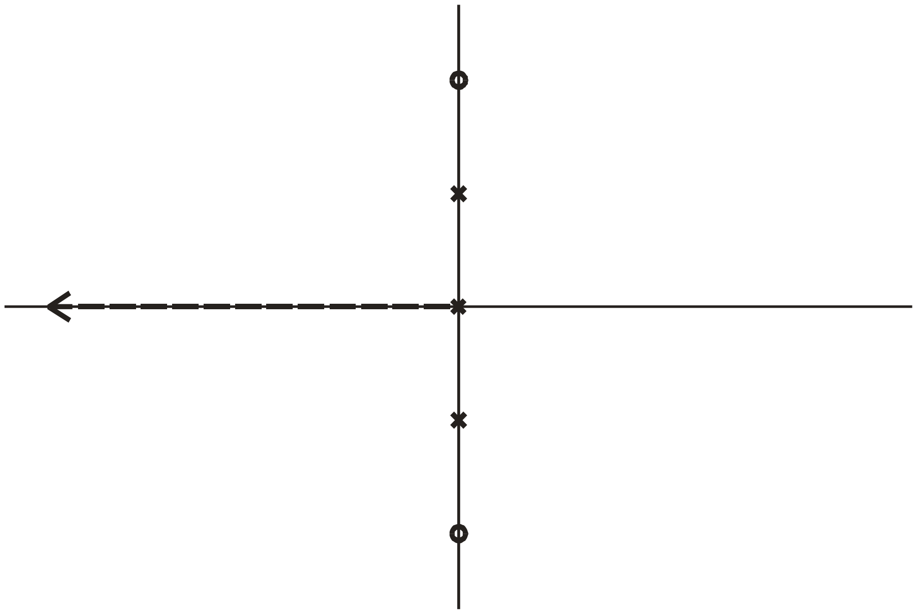
$$\omega = 1.38 \qquad\qquad K = 4.5$$

Finally fill in the locus.



5.5f) First determine the pole and zero locations. There are 3 poles at 0, $+i$, $-i$ and respectively, and there are two zeros at $+2i$, $-2i$ respectively.

Now the loci on the real axis. The rule is if the #poles + #zeros to the right of a point on the real axis is odd then that point is on the locus.



There will be only one asymptote and the centroid will be at the origin.

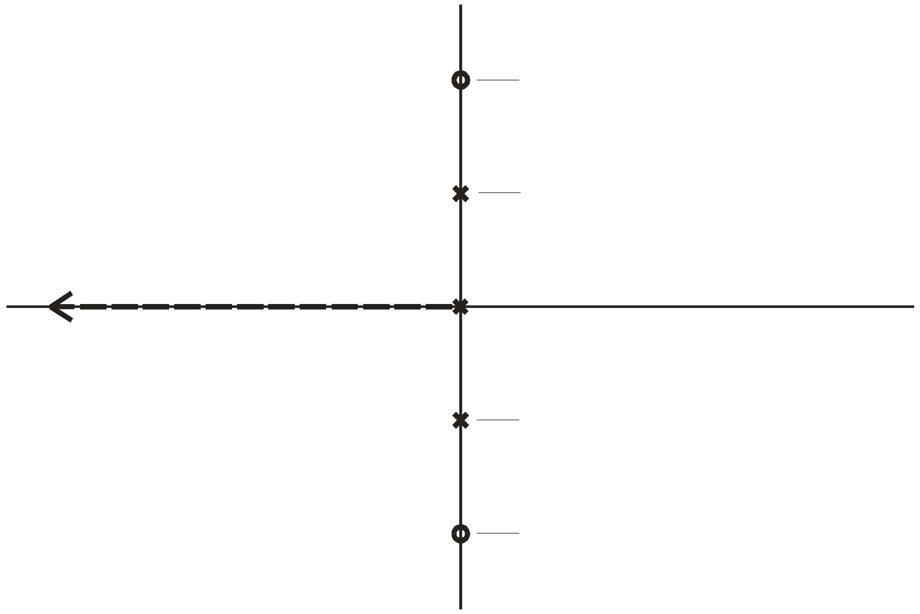
By inspection it can be seen that the pole at $0+0i$ will depart at 180°. From the pole located at $+i$ the departure angle is calculated as follows:

$$
\begin{aligned}
\phi_{dep} &= \sum \psi_i - \sum \phi_i - 180^{\circ} \\
&= +90^{\circ} + \left(-90^{\circ}\right) - 90^{\circ} - 90^{\circ} - 180^{\circ} \\
&= 0
\end{aligned}
$$

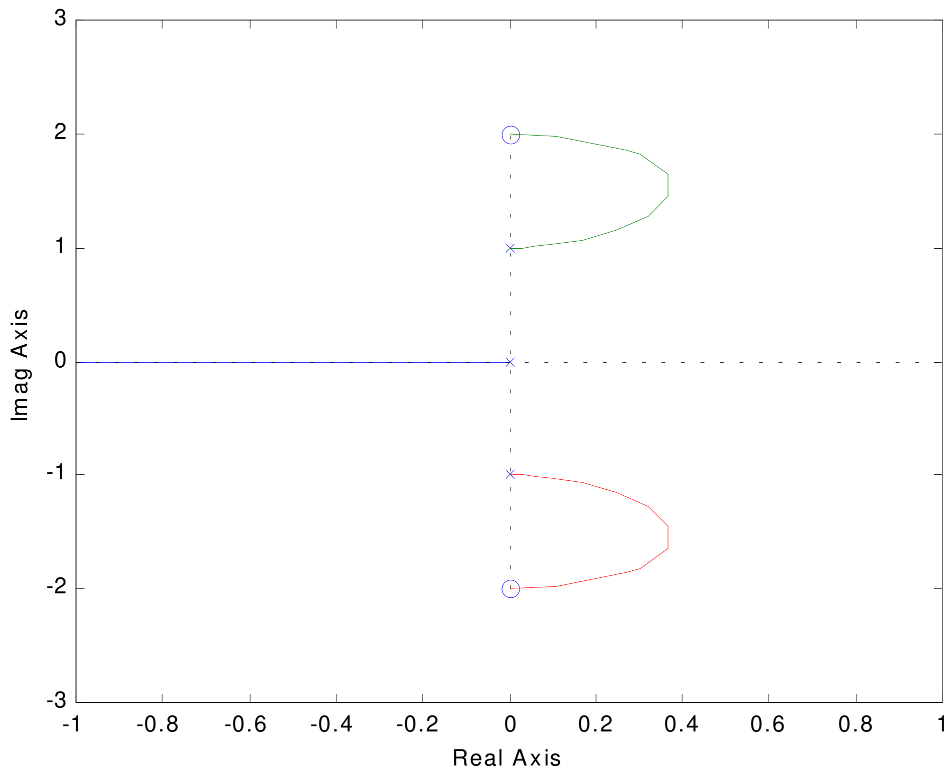
And by symmetry the departure angle from the pole at $-i$ is 0.

From the zero located at $+2i$ the arrival angle is calculated as follows:

$$
\begin{aligned}
\psi_{arr} &= \sum \phi_i - \sum \psi_i + 180^{\circ} \\
&= +90^{\circ} + 90^{\circ} + 90^{\circ} - 90^{\circ} + 180^{\circ} \\
&= 0
\end{aligned}
$$



Finally fill in the locus.



5.9) The transfer function for the system in figure 5.59 is:

$$H(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}$$
$$G(s) = \frac{9}{s^2}$$
$$C(s) = \frac{b + 16s}{b + s}$$

Thus the characteristic equation is given by:

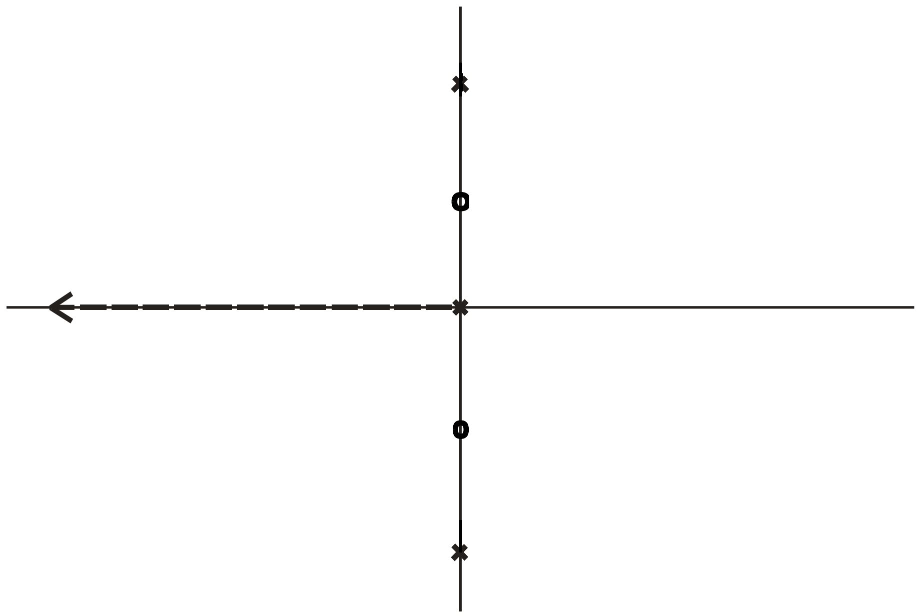
$$0 = s^3 + bs^2 + 144s + 9b$$

For the purpose of drawing a root locus with respect to parameter b manipulate this equation until it matches the form $1 + KT(s) = 0$.

$$0 = b(s^2 + 9) + s^3 + 144s$$
$$= 1 + b\frac{s^2 + 9}{s^3 + 144s}$$

There are 3 poles at 0,12i, -12i and respectively, and there are two zeros at $+3i, -3i$ respectively

Case I: $0 < b < \infty$



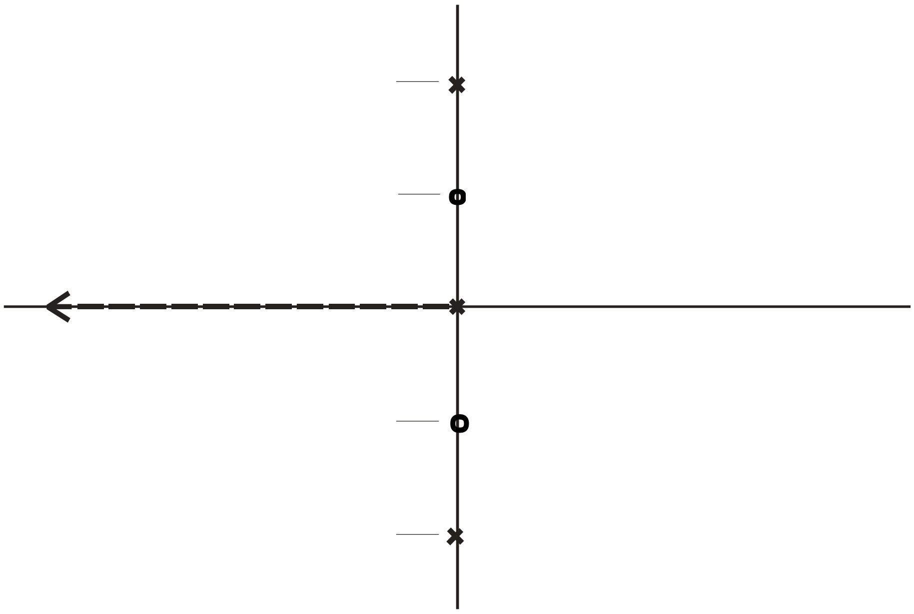
By inspection it can be seen that the pole at $0+0i$ will depart at 180°. From the pole located a +12i the departure angle is calculated as follows:

$$\begin{aligned}\phi_{dep} &= \sum \psi_i - \sum \phi_i - 180^{\circ} \\ &= +90^{\circ} + \quad 90^{\circ} \ -90^{\circ} - 90^{\circ} - 180^{\circ} \\ &= \text{-}180\end{aligned}$$

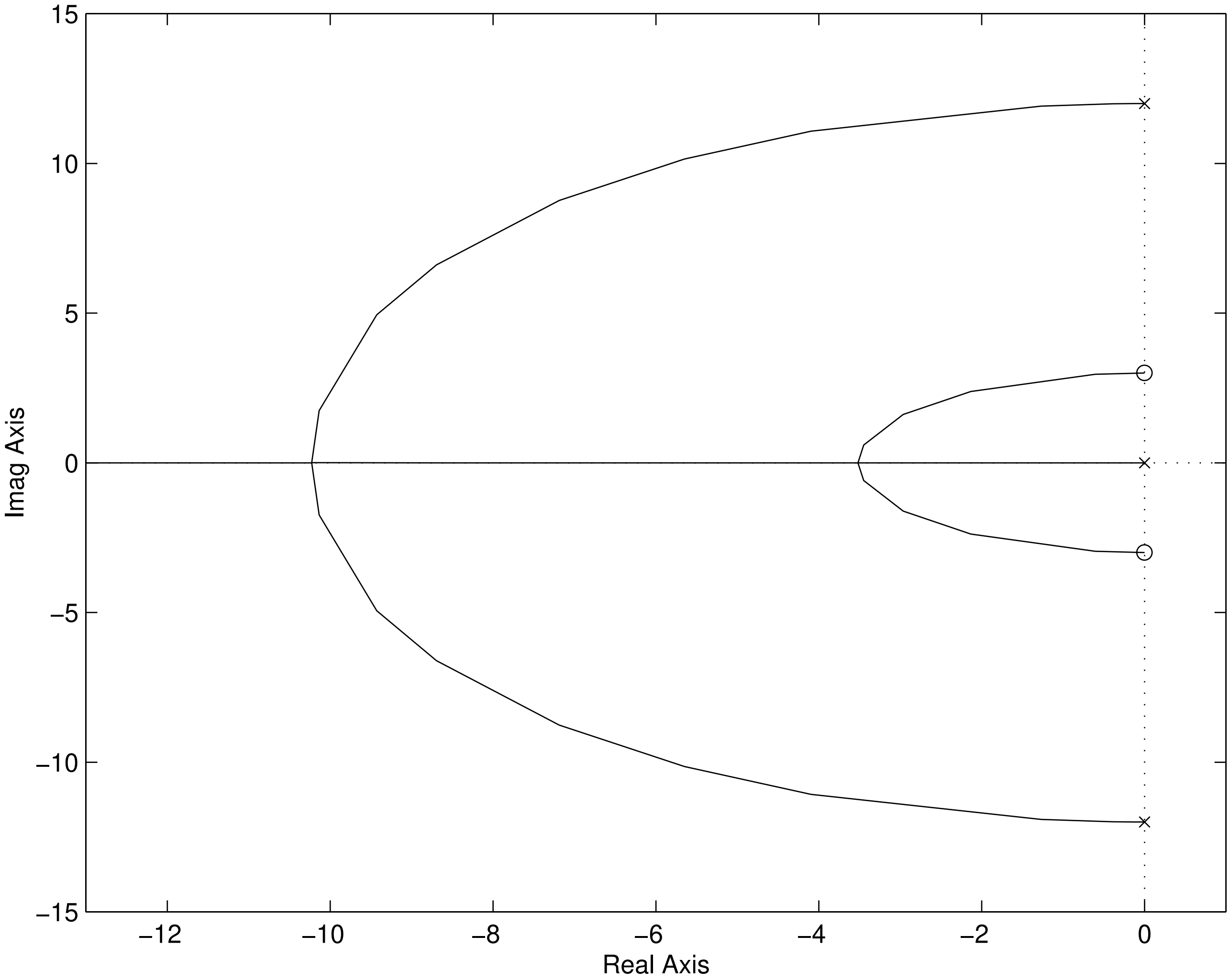
And by symmetry the departure angle from the pole at -12i is -180.

From the zero located at $+3i$ the arrival angle is calculated as follows:

$$\begin{aligned}\psi_{arr} &= \sum \phi_i - \sum \psi_i + 180^{\circ} \\ &= -90^{\circ} + 90^{\circ} + 90^{\circ} - 90^{\circ} + 180^{\circ} \\ &= 180\end{aligned}$$



Finally fill in the locus.



15
10
5
0
−5
−10
−15
Imag Axis
−12
−10
−8
−6
−4
−2
0
Real Axis

Case II: $-\infty < b < 0$

In this case a zero degree locus must be drawn. By following the rules for drawing a zero degree locus it is easy to see that the plot for Case II will be a mirror image about the imaginary axis. Here is the MATLAB plot.