Problem5.14 : The system's characteristic equation is:

$$s^2 + s + 1 + \alpha(s + 1) = 0$$

which implies that

$$1 + \alpha \frac{s + 1}{s^2 + s + 1} = 0$$

When $\alpha \geq 0$, the plant is stable and the corresponding $180°$ root locus is shown in Figure 1.

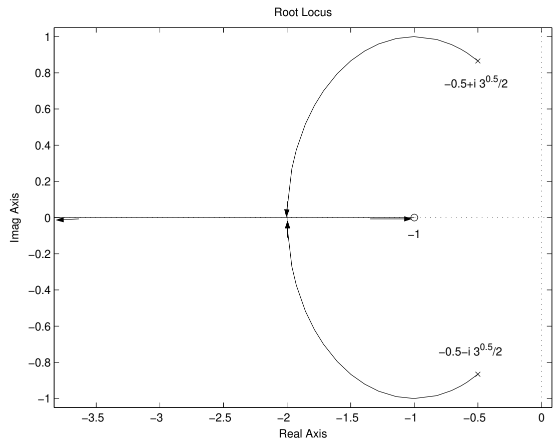

Figure 1: Prob. 5.14

When $\alpha < 0$, variations in α can result in an unstable system. The $0°$ root locus is shown in Figure 2.

From root locus we can see that the root locus crosses the $j\omega$ axis at $s = 0$ and

$$\begin{aligned} \alpha &= -\frac{s^2 + s + 1}{s + 1}\Big|_{s=0} \\ &= -1 \end{aligned}$$

which implies that $\alpha \leq -1$ results in an unstable system.
We can also use Routh criterion to verify it. The Routh array is shown as follows,

$$\begin{array}{lll} s^2 & 1 & 1 + \alpha \\ s^1 & 1 + \alpha & 0 \\ s^0 & 1 + \alpha & 0 \end{array}$$

and in order to make the system stable, we must have $1 + \alpha > 0$, which implies that $\alpha \leq -1$ will result in an unstable system.

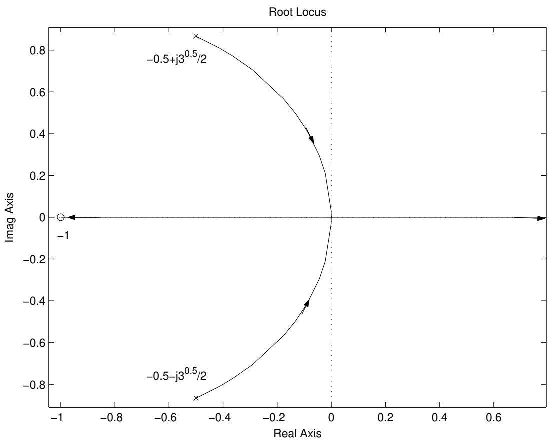

Figure 2: Prob. 5.14

Problem5.25 : We consider $D(s)$ in different cases.
(a) If we choose $D(s) = K$, then

$$K = -\frac{1}{G(s)}\Big|_{s=-1+\sqrt{3}j} = -s(s+1)\Big|_{s=-1+\sqrt{3}j} = 3 + \sqrt{3}j$$

Since K must be real, this specification cannot be achieved by $D(s) = K$.
(b) If we choose lead compensation:

$$D(s) = K\frac{s+z}{s+p} \qquad (p > z)$$

Dominant closed-loop poles are: $s = -1 \pm \sqrt{3}j$.
We will choose a reasonable location for the zero and then calculate the associated pole position which forces the locus through the point $s = -1 + j\sqrt{3}$. Pick $z = 2$, by condition $\angle D(s) + \angle G(s) = -180°$, we have

$$-\theta_p + 60° - 90° - 120° = -180°$$

which implies that $\theta_p = 30°$.
Let $p = x + 1$:

$$tan\theta_p = \frac{\sqrt{3}}{x} \Rightarrow x = 3 \Rightarrow p = 4$$

Gain K :

$$\begin{aligned} K &= -\frac{1}{D(s)G(s)}\Big|_{s=-1+\sqrt{3}j} \\ &= -\frac{s(s+1)(s+4)}{s+2}\Big|_{s=-1+\sqrt{3}j} \\ &= 6 \end{aligned}$$

Thus finally we get

$$D(s) = 6\frac{s+2}{s+4}$$

Problem5.39a : Using the lead compensation:

$$G(s)H(s) = \frac{K(s+2)}{s^2(s+4)}$$

The root locus is shown in Figure 3.

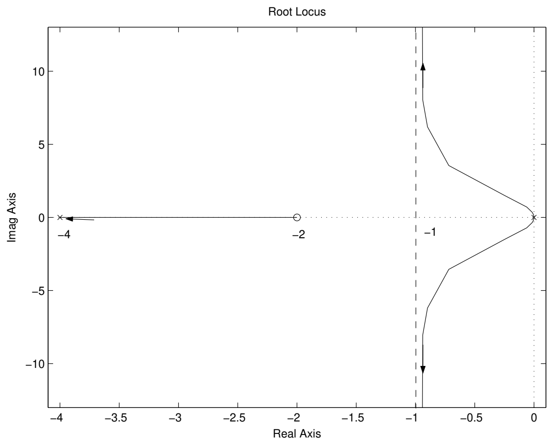

Figure 3: Prob. 5.39a

From root locus we can see that the system is stable for all $K > 0$. Note that with constant feedback only (no compensation), the characteristic equation is

$$s^2 + K = 0$$

which has roots on the $j\omega$-axis only for all $K > 0$.

Problem5.45 :
(a) If we use propotional control, then

$$G(s)D(s) = \frac{K}{s(s+1)}$$

The root locus is given in Figure 4. From the root locus, roots with $\zeta = 0.7$ are located at $s = -0.5 \pm j0.51$.

$$K = |s_0||s_0+1|\Big|_{s+0=-0.5+j0.510} = \frac{(0.714)^2}{1} = 0.51$$

The disturbance response is:

$$y(\infty) = \lim_{s\to 0}\Big(\frac{Y(s)}{W(s)}\Big)W(s) = \lim_{s\to 0} s\Big[\frac{1}{s^2+s+0.51}\Big]\Big(\frac{1}{s}\Big) = 1.96 > \frac{4}{5}$$

Thus, propotional control is not adequate.

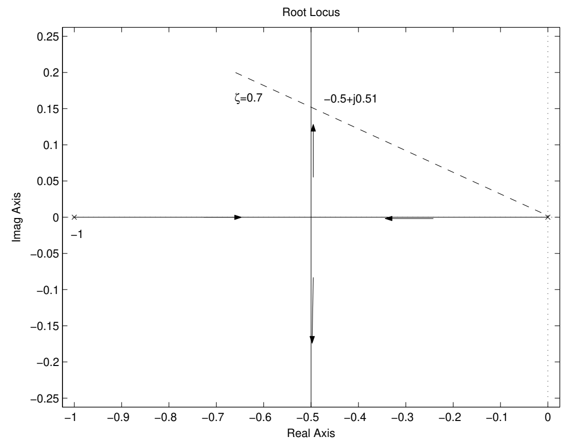

Figure 4: Prob. 5.45 (a)

(b) Let $D(s) = K + K_d s$, the characteristic equation is:

$$s^2 + s + K + K_d s = 0$$

Now, we have

$$y(\infty) = \lim_{s\to 0} s\Big[\frac{1}{s^2+s+K+K_d s}\Big]\Big(\frac{1}{s}\Big) = \frac{1}{K}$$

Thus, fix K with $K > \frac{5}{4}$, we get $y(\infty) < \frac{4}{5}$.
Next, plot root locus for varying K_d (with the fixed K) until poles with $zeta = 0.7$ are obtained. (See part (c)) That is, plot root locus for the form:

$$1 + \frac{K_d s}{s^2+s+K} = 0$$

(c) Let $K = \frac{6}{4} = 1.5$, we have $y(\infty) = \frac{4}{6} < \frac{4}{5}$.
The characteristic equation is:

$$1 + K_d\frac{s}{(s+0.5+1.18j)(s+0.5-1.18j)} = 0$$

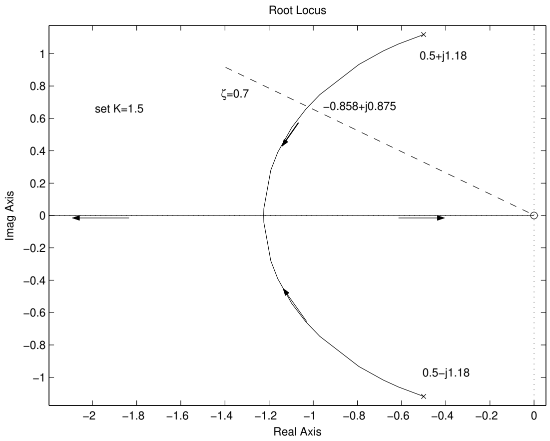

Figure 5: Prob. 5.45 (b)

From the root locus (see Figure 5), we have

$$
\begin{aligned}
K_d &= \left.\frac{|s_0 + 0.5 + 1.18j||s_0 + 0.5 - 1.18j|}{|s_0|}\right|_{s_0=-0.858+0.875j} \\
&= \frac{(0.432)(2.024)}{1.225} \\
&= 0.714
\end{aligned}
$$